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An Algorithm for Cavity Reconstruction in Electrical Impedance Tomography*

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Abstract: We consider the inverse problem of finding cavities within some object from electrostatic measurements on the boundary. By a cavity we understand any object with a different electrical conductivity from the background material of the body. We give an algorithm for solving this inverse problem based on the output nonlinear least-square formulation and the regularized Newton-type iteration. In particular, we present a number of numerical results to highlight the potential and the limitations of this method.

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1 Introduction

Electrical impedance tomography (EIT) is a technique to recover spatial properties of the interior of a conducting object from electrostatic measurements taken on its boundary, an important task in nondestructive testing. The problem has important applications in medical imaging, geophysics and environmental sciences. The technology is rapidly advancing, surveys include (see, e.g., [1]–[4]).

Avid mathematical interest in EIT originated by Calderón[5] in 1980, and its unique solvability for isotropic, i.e., scalar, conductivities of a wide class was obtained in there and higher space dimensions by Sylvester and Uhlmann[6] in 1987 and in two dimensions by Nachman[7] in 1996. Recently, uniqueness in two dimensions was shown by Astala and Päivärinta[8]. Currently, the sharpest results for three dimensions and higher is made by Brown and Torres[9]. On the other hand, it is well known that the inverse problem of EIT is not uniquely solvable without the isotropy assumption (see, e.g., [10]).

The reconstruction methods of EIT from boundary measurements, Calderón suggested in a specific linearization method which assumes that the conductivity is close to a constant.

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Other examples of linearization-based algorithms include backprojection methods (see, e.g., [11]), moment methods (see, e.g., [12]), and one-step Newton methods (see, e.g., [13]). Algorithms solving the full nonlinear problem have been iterative in nature, with the exception of some other direct methods including layer-stripping (see, e.g., [14]), D-bar methods (see, e.g., [15]), the Factorization method (see, e.g., [2]) and the probe method (see, e.g., [16]) together with their variants.

Unfortunately, the relationship between the boundary current-voltage data and the internal conductivity distribution bears a nonlinearity and low sensitivity, in mathematical terms which means that EIT is an ill-posed problem. For this reason, it is important to incorporate as much a priori knowledge about the object as possible, and this is the reason why the cavity problem may be somewhat easier to approach and more likely to eventually solve numerically. In the early 1990s, magnetic resonance electrical impedance tomography (MREIT) was proposed to deal with the difficulties of EIT. The key idea of MREIT was based on measuring magnetic flux density $\boldsymbol{B}$ by using a current-injection MRI technique, and the inverse problem is divided into two categories: using current density $\boldsymbol{J}$ for image reconstructions called $\boldsymbol{J}$-based MREIT; and the other is B_z-based MREIT (see, e.g., [17]).

In this paper, we consider the cavity problem in EIT, i.e., we give an algorithm to determine the shape of a cavity within the object from electrostatic measurements on the boundary. It is organized as follows: in Section 2, we formulate the problem, and transform the cavity problem into a nonlinear operator equation using coordinate transformation and analytic continuation. Section 3 presents an iterative algorithm based on the output nonlinear least-squares formulation and the regularized Newton-type method, and Section 4 contains several numerical examples and concluding remarks.

2 Mathematical Formulation of the Problem

We consider a simply connected domain

$$B_R = \{(x, y) \mid x^2 + y^2 < R^2\} \subset \mathbf{R}^2$$

with boundary ∂B_R. Assume that the object is homogeneous and conducting except for a insulating cavity Ω, which is convex with boundary $\partial\Omega$. And assume that there exists a constant $r_0 > 0$ satisfying

$$B_{r_0} = \{(x, y) \mid x^2 + y^2 \leq r_0^2\} \subset \bar{\Omega} \subset B_R.$$

∂B_R and $\partial\Omega$ are considered to be sufficient smooth with $\mathbf{n}$ being the outer (relative to $B_R \backslash \bar{\Omega}$) unit normal vector.

There is a prescribed boundary current:

$$f \in L_0^2(\partial B_R) = \Big\{ f \in L^2(\partial B_R) : \int_{\partial B_R} f(s) \mathrm{d}s = 0 \Big\},$$

where

$$L^2(\partial B_R) = \Big\{ f(s) : \int_{\partial B_R} \mid f(s)|^2 \mathrm{d}s < +\infty \Big\}.$$

The corresponding boundary potential $g \in L_0^2(\partial B_R)$ can be measured without physical