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Electromagnetic Scattering in a Two-layered Medium*

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Abstract: The object of this paper is to investigate the three-dimensional electromagnetic scattering problems in a two-layered background medium. These problems have an important application in today's technology, such as to detect objects that are buried in soil. Here, we model both the exterior impedance problem and the inhomogeneous medium problem in $\mathbf{R}^3$. We establish uniqueness and existence for the solution of the two scattering problems, respectively.

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1 Introduction

Recently, considerable attention has been devoted to the analysis for the electromagnetic scattering problems in a layered medium (see [1]–[10]). These problems have an important application in today's engineering and physics, such as to detect objects that are buried in soil. Here, we are interested in the three-dimensional electromagnetic scattering problems in a two-layered background medium. Among the extensive literatures we refer to the publications [1], [3] and [7]. The paper [3] is concerned with the solution of Maxwell equations in the modeling of the scattering of a time-harmonic electromagnetic wave by a perfect conductor (obstacle) located in a two-layered medium. The use of the Silver-Müller radiation condition in each layer was shown to provide a well-posed scattering problem. The analysis was based on the study of the Green tensor. The analyticity properties of the scattering problem with respect to the frequency were also investigated. In the paper [7], the author studied also the electromagnetic scattering problems in a two-layered medium from a perfectly conducting

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obstacle. In contrast with the paper [3], in one layer the Silver-Müller radiation condition was used, and in the other layer an exponential decay condition was used. The scattered field was modeled via a boundary layer approach and for its kernel the Green's matrix for the two-layered medium was constructed. The author established uniqueness and existence for the solution of the corresponding boundary integral equation. In this paper, we are concerned with both the exterior impedance problem and the inhomogeneous medium problem in a two-layered background medium. We establish uniqueness and existence for the solution of the two scattering problems, respectively. The idea of our proof is inspired by the paper [7].

We model the obstacle (or inhomogeneous part) by some domain D in the lower half-space. For simplicity, we assume that D has a sufficiently smooth boundary, i.e., we assume a C^2 boundary. However, in principle, D can be a domain with corners and edges. We denote by $\mathbf{D}_1 = D_1 = \{x = (x_1, x_2, x_3) \in \mathbf{R}^3 : x_3 > 0\}$ the upper half-space and by $\mathbf{D}_2 = \{x = (x_1, x_2, x_3) \in \mathbf{R}^3 : x_3 < 0\}$ the lower half-space. $S = \{x = (x_1, x_2, x_3) \in \mathbf{R}^3 : x_3 = 0\}$ denotes the interface between $\mathbf{D}_1$ and $\mathbf{D}_2$. We assume $D \subset \mathbf{D}_2$ to be a bounded domain with connected complement and define $D_2 = \mathbf{D}_2 \backslash \bar{D}$.

We consider the propagation of electromagnetic waves with frequency ω in the two-layered medium consisting of the isotropic half spaces $\mathbf{D}_j$ with electric permittivity ϵ_j, magnetic permeability μ_j and electric conductivity σ_j for $j = 1, 2$. First, we describe the exterior impedance problem. The electromagnetic wave is described by the electric fields $\mathscr{E}_j$ and the magnetic fields $\mathscr{H}_j$ in D_j that satisfy the Maxwell equations

$$\mathrm{curl}\mathscr{E}_j + \mu_j \frac{\partial \mathscr{H}_j}{\partial t} = 0, \qquad \mathrm{curl}\mathscr{H}_j - \epsilon_j \frac{\partial \mathscr{E}_j}{\partial t} = \sigma_j \mathscr{E}_j.$$

We consider the harmonic time-dependence in the form

$$\mathscr{E}_j(x,t) = \mathrm{Re}\Big\{ \Big(\epsilon_j + \frac{\mathrm{i}\sigma_j}{\omega}\Big)^{-1/2} E_j(x) \mathrm{e}^{-\mathrm{i}\omega t} \Big\}, \qquad \mathscr{H}_j(x,t) = \mathrm{Re}\{(\mu_j)^{-1/2} H_j(x) \mathrm{e}^{-\mathrm{i}\omega t}\}. \quad (1.1)$$

Then the space-dependent parts $E_j, H_j \in C^1(D_j) \cap C(\bar{D}_j)$, $j = 1, 2$, satisfy the time-harmonic Maxwell equations in the symmetric form

$$\mathrm{curl}E_j - \mathrm{i}k_j H_j = 0, \quad \mathrm{curl}H_j + \mathrm{i}k_j E_j = 0 \qquad \text{in } D_j \quad (1.2)$$

with wave numbers

$$k_j = \sqrt{\Big(\epsilon_j + \frac{\mathrm{i}\sigma_j}{\omega}\Big)\mu_j \omega^2}, \qquad j = 1, 2. \quad (1.3)$$

The square roots for the wave numbers are chosen so that $\mathrm{Re}k_j \geq 0$ and $\mathrm{Im}k_j \geq 0$, $j = 1, 2$. For some applications, such as in state of the air-soil, for $\mathbf{D}_1$, we have air which is non-conducting, i.e., $\sigma_1 = 0$, and consequently, $\mathrm{Im}k_1 = 0$. However, in $\mathbf{D}_2$ we have soil with a conductivity $\sigma_2 > 0$, and therefore $\mathrm{Im}k_2 > 0$. According to the continuity of the tangential components of the electric field $\mathscr{E}_j$ and the magnetic field $\mathscr{H}_j$ cross the interface, we can show that at the interface between $\mathbf{D}_1$ and $\mathbf{D}_2$, the fields satisfy the transmission conditions

$$\nu \times E_1 = a_E \nu \times E_2, \quad \nu \times H_1 = a_H \nu \times H_2 \qquad \text{on } S \quad (1.4)$$

with the constants a_E and a_H given by

$$a_E = \sqrt{\frac{\epsilon_1}{\epsilon_2 + \frac{\mathrm{i}\sigma_2}{\omega}}}, \qquad a_H = \sqrt{\frac{\mu_1}{\mu_2}}, \quad (1.5)$$