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New Jacobi Elliptic Function Solutions for the Generalized Nizhnik-Novikov-Veselov Equation*

HONG BAO-JIAN
(*Department of Basic Courses, Nanjing Institute of Technology, Nanjing, 211167*)

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Abstract: In this paper, a new generalized Jacobi elliptic function expansion method based upon four new Jacobi elliptic functions is described and abundant solutions of new Jacobi elliptic functions for the generalized Nizhnik-Novikov-Veselov equations are obtained. It is shown that the new method is much more powerful in finding new exact solutions to various kinds of nonlinear evolution equations in mathematical physics.

Key words: generalized Jacobi elliptic function expansion method, Jacobi elliptic function solution, exact solution, generalized Nizhnik-Novikov-Veselov equation
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1 Introduction

In recent years, due to the wide applications of soliton theory in natural science, searching for exact soliton solutions of nonlinear evolution equations plays an important and significant role in the study on the dynamics of those phenomena (see [1]). Particularly, various powerful methods have been presented, such as inverse scattering transformation, Cole-Hopf transformation, Hirota bilinear method, homogeneous balance method, Backlund transformation, Darboux transformation, projective Riccati equations method and so on. In this paper, we discuss a generalized Nizhnik-Novikov-Veselov (GNNV) equation by our generalized Jacobi elliptic function expansion method (see [2]) proposed recently. As a result, more new exact solutions are obtained. The character feature of our method is that, without much extra effort, we can get series of exact solutions by using a uniform way. Another advantage of our method is that it also applies to general higher-dimensional nonlinear partial differential equations.

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We consider the following GNNV equations (see [3–6]):

$$\begin{cases} u_t + au_{xxx} + bu_{yyy} + cu_x + du_y - 3a(uv)_x - 3b(uw)_y = 0, \\ u_x - v_y = 0, \\ u_y - w_x = 0, \end{cases} \tag{1.1}$$

where a, b, c and d are arbitrary constants. For

$$c = d = 0,$$

the GNNV equations (1.1) are degenerated to the usual two-dimensional NNV equations (see [7–8]), which is an isotropic Lax extension of the classical (1+1)-dimensional shallow water-wave KdV model. When

$$a = 1, \qquad b = c = d = 0,$$

we get the asymmetric NNV equation, which may be considered as a model for an incompressible fluid. Some types of exact solutions of the GNNV equations have been studied in recent years (see [9–13]).

2 Summary of the New Generalized Jacobi Elliptic Functions Expansion Method

Given a partial differential equation with three variables x, y and t

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, u_{yt}, u_{xy}, \cdots) = 0, \tag{2.1}$$

we seek the following formal solutions of the given system by a new intermediate transformation:

$$u(\xi) = \sum_{i=0}^{n} A_i F^i(\xi) + \sum_{\substack{i,j=1 \\ i\le j\le n}}^{n} [B_i F^{j-i}(\xi)E^i(\xi) + C_i F^{j-i}(\xi)G^i(\xi) + D_i F^{j-i}(\xi)H^i(\xi)], \tag{2.2}$$

where A_0, A_i, B_i, C_i, D_i $(i = 1, 2, \cdots, n)$ are constants to be determined later, $\xi = \xi(x, y, t)$ is an arbitrary function with the variables x, y and t, the parameter n can be determined by balancing the highest order derivative terms with the nonlinear terms in (2.1), and $E(\xi)$, $F(\xi)$, $G(\xi)$, $H(\xi)$ are the arbitrary arrays of the four functions

$$e = e(\xi), \quad f = f(\xi), \quad g = g(\xi), \quad h = h(\xi)$$

respectively. The selection obeys the principle which makes the calculation more simple. We ansatz

$$\begin{cases} e = \dfrac{1}{p + q\mathrm{sn}\xi + r\mathrm{cn}\xi + l\mathrm{dn}\xi}, \\ f = \dfrac{\mathrm{sn}\xi}{p + q\mathrm{sn}\xi + r\mathrm{cn}\xi + l\mathrm{dn}\xi}, \\ g = \dfrac{\mathrm{cn}\xi}{p + q\mathrm{sn}\xi + r\mathrm{cn}\xi + l\mathrm{dn}\xi}, \\ h = \dfrac{\mathrm{dn}\xi}{p + q\mathrm{sn}\xi + r\mathrm{cn}\xi + l\mathrm{dn}\xi}, \end{cases} \tag{2.3}$$