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Analysis of Bifurcation and Stability on Solutions of a Lotka-Volterra Ecological System with Cubic Functional Responses and Diffusion*

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Abstract: This paper deals with a Lotka-Volterra ecological competition system with cubic functional responses and diffusion. We consider the stability of semi-trivial solutions by using spectrum analysis. Taking the growth rate as a bifurcation parameter and using the bifurcation theory, we discuss the existence and stability of the bifurcating solutions which emanate from the semi-trivial solutions.

Key words: Lotka-Volterra ecological system, stability, bifurcating solution

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1 Introduction

It is one of the elementary concerns of many researchers that analyze the dynamics of biological populations by reaction-diffusion equations. During the past decades, intensive studies in pursuing the ecological systems with various boundary conditions derived from interacting processes of several species have been investigated mathematically. These systems, such as the Lotka-Volterra models (see [1–7]), Leslie-Gower models (see [8–10]), Sel'kov models (see [11–13]) and Brusselator models (see [14–16]) are important research branches. In these references, the authors discussed different ecological models with various boundary conditions. They analyzed the dynamical behavior of these models in different ways, including the

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existence, nonexistence, boundedness, bifurcation, the stability and some other characters of positive solutions to these models, and many valuable and classical results were obtained.

Among numerous literatures on Lotka-Volterra models, the reaction terms of quadratic are relatively common. In the present paper, we investigate the following Lotka-Volterra competition reaction-diffusion system with cubic functional responses:

$$\begin{cases} u_t - d_1\Delta u = au - bu^3 - cuv^2, & x \in \Omega,\ t > 0, \\ v_t - d_2\Delta v = ev - fu^2v - gv^3, & x \in \Omega,\ t > 0, \\ u = v = 0, & x \in \partial\Omega,\ t > 0, \\ u = u_0 \geq 0\ (\not\equiv 0), \quad v = v_0 \geq 0\ (\not\equiv 0), & x \in \Omega,\ t = 0, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbf{R}^N$ is an open, bounded domain with smooth boundary $\partial\Omega$, $u = u(x,t)$ and $v = v(x,t)$ are the population densities of the two competing species, d_1 and d_2 are the diffusion coefficients of u and v, a and e represent their respective birth rates, b and g account for the self-regulation of each species, and c and f describe the competition between the two species. All the parameters are positive constants. The homogeneous boundary condition means that the habitat Ω where the two species live is surrounded by a hostile environment. With these interpretations, only solutions of (1.1) with u and v nonnegative are physically of interest.

Biologically, we can interpret this system as follows. The functions $a - bu^2$, fu^2, $e - gv^2$ and cv^2 describe how species u and v interact among themselves and with each other. Firstly, the case $f > b$ and $c > g$ means that the species u interacts strongly with species v and weakly among themselves. Similarly, for species v, they interact more strongly with u than they do with themselves. Hence, when $f > b$ and $c > g$, the equations in (1.1) model a highly competitive system. Secondly, the opposite situation happens when $f < b$ and $c < g$, namely, both species interact more strong among themselves than they do with the other species. So, when $f < b$ and $c < g$, the equations in (1.1) model a weakly competitive system. Thirdly, when both $f = b$ and $c = g$, each species interacts with the other almost at the same rate with that they interact among themselves. If $a = e$, this can be interpreted as the maximum relative growth rates being the same for both species.

If we only consider the case that u and v are functions of x alone, then it is natural to look for the steady-state solutions of (1.1). Furthermore, if both components of such a solution are strictly positive, it is referred to as a coexistence state. The main aim of this paper is to study the bifurcation and stability of the steady-state solutions of the system (1.1), that is, to study the bifurcation and stability of the classical solutions of the following elliptic system:

$$\begin{cases} -d_1\Delta u = au - bu^3 - cv^2u, & x \in \Omega, \\ -d_2\Delta v = ev - fu^2v - gv^3, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega. \end{cases} \tag{1.2}$$

The organization of this paper is as follows. In Section 2, by using the method of spectrum analysis, we first give the stability of the semi-trivial solutions of the system. In Section 3, by the bifurcation theory, we discuss the existence and stability of the bifurcating solutions