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Trivariate Polynomial Natural Spline for 3D Scattered Data Hermit Interpolation*

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Abstract: Consider a kind of Hermit interpolation for scattered data of 3D by trivariate polynomial natural spline, such that the objective energy functional (with natural boundary conditions) is minimal. By the spline function methods in Hilbert space and variational theory of splines, the characters of the interpolation solution and how to construct it are studied. One can easily find that the interpolation solution is a trivariate polynomial natural spline. Its expression is simple and the coefficients can be decided by a linear system. Some numerical examples are presented to demonstrate our methods.

Key words: scattered data, Hermit interpolation, natural spline

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1 Introduction

Scattered data fitting is used widely in many fields such as data compressing, automobile shape designing, ship lofting, aerofoil and airframe designing, fashion designing, geologic ore-exploring, medical image processing and so on. So it is one of the most important problems (see [1–4]). Since 1960s, many researchers have been paying more attention to scattered data fitting for curves and surfaces and have presented different methods. Moreover, point cloud data fitting of 3D have been studied deeply and widely in recent years (see [5–7]).

By the tensor product method of curves, the problem of scattered data interpolation can be solved when scattered points are located on some grid regularly. But generally, scattered data, which is obtained from sampling survey, is not regular and the tensor product method

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of curves cannot be used. So non-tensor product methods need to be constructed for solving scattered data interpolation problems. Now, there are many different non-tensor product approaches for scattered data interpolation, such as natural neighbor methods, Shepard methods, Kriging methods, level B-spline methods, thin plane spline methods, radial basis function methods and so on (see [8]). Till now, many researchers still pay attention to the problem of scattered data fitting, and some new methods have been given. Lai[9] and Wu[10] have done some works to sum up this methods in their literature. But unlike unvariate B-spline of degree three, which has a series good properties and can be used to solve unvariate scattered data interpolation perfectly, the solutions of the problem for large scattered data fitting and multivariate interpolation are still not perfect.

In 1972, Laurent[11] summed up unvariate polynomial natural spline interpolation for scattered data and proposed variational theory of spline in Hilbert spaces in 1D cases. Since 1980s, Li *et al.*[12] have studied in this fields. They tried to generalize the methods, which are used to solve unvariate scattered data interpolation by polynomial natural splines, to bivariate cases by variational theory of splines in Hilbert spaces. They provided bivariate polynomial natural splines for scattered data and studied optimal multivariate interpolation for scattered data problem with continuous boundary conditions and discrete boundary conditions on rectangle domain in general blending spline space. Chui and Guan[13] generalized the results of bivariate to general multivariate completely. Guan[14] also studied local supported basis which is similar to B-spline basis. In 2003, the computing methods for the properties of the local supported basis and interpolating natural spline were published in [15]. However, since its objective functional is expressed with a series integral terms (see [16]), it is so complicated that cannot be used perfectly and the interpolation results are impacted by the number of interpolatory points on the boundary of the domain. In 2001, Bezhaev and Vasilenko[17] summed up the variational theory of spline in Hilbert spaces and their applications for scattered data fitting in multi-dimensional cases, but the solutions are not explicit in most cases. Recently, Guan *et al.*[18] have improved the methods and presented a new kind of bi-cubic interpolating natural spline for 2D scattered data. Its objective functional is very simple, has no discrete boundary conditions and can be used perfectly. But this method is a simple interpolation, in other words, interpolating some functions only use their values on scattered points.

In 3D animation, medical image precessing and some other fields, 3D scattered data interpolation is used usually. So it is important to solve the interpolation problem for 3D scattered data. In order to make the interpolation function become smooth enough, for example, let the interpolation function belong to a $C^1(\Omega)$ space, we need Hermit interpolation sometimes. But for scattered data Hermit interpolation, its construction is more difficult usually.

In this paper, to deal with Hermit interpolation for 3D scattered data, a kind of trivariate polynomial natural splines method is presented. The interpolation solution σ is

$$\sigma = \arg \min_{u \in X, Au = z} \{\|Tu\|_Y^2\}, \tag{1.1}$$

where $\|Tu\|_Y^2$ is an energy functional and $Au = z$ is the interpolating condition (see cor-