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General Structures of Block Based Interpolational Function*

Zou Le[1] and Tang Shuo[2]
(*1. Key Lab of Network and Intelligent Information Processing, Hefei University, Hefei, 230601*)
(*2. School of Mathematics, Hefei University of Technology, Hefei, 230009*)

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Abstract: We construct general structures of one and two variable interpolation function, without depending on the existence of divided difference or inverse differences, and we also discuss the block based osculatory interpolation in one variable case. Clearly, our method offers many flexible interpolation schemes for choices. Error terms for the interpolation are determined and numerical examples are given to show the effectiveness of the results.
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1 Introduction

Kahng[1] showed the generalizations of univariate Newton's method and applied it to the approximation problems in 1967. These generalizations extend the applicable interpolation functions from polynomials to rational functions, their transformations and some nonlinear functions. Also, these generalizations enable us to treat the interpolation in a unified manner. Furthermore, Kahng[2] described a class of interpolation functions and showed the explicit method of osculatory interpolation with a function in the class in 1969. These two functions have many special cases, such as Newton interpolation polynomial, Thiele-type continued fraction interpolation, Hermite interpolation, Salzer-type osculatory interpolation, trigonometric functions interpolations and so on. In 1999, by introducing multiple param-

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eters, Tan and Fang[3] studied several general frames for bivariate interpolation which include many classical interpolant scheme, such as bivariate Newton interpolation, Thiele-type branched continued fractions for two variables, Newton-Thiele's blending rational interpolation, Thiele-Newton's blending rational interpolation, and symmetric branched continued fraction discussed by Cuyt[4] and Murphy *et al.*[5] Recently, Tang and Zou[6] have improved and extended the general frames studied by Tan and Fang by introducing multiple parameters, so that the new frames can be used to deal with the interpolation problems where inverse differences are nonexistent or unattainable points occur. Tan[7] discussed the more general interpolation grids. In this paper, we construct block based interpolation structure, which are extensions and improvements of the above frames.

In Section 2, we discuss the general structure of univariate interpolant function and block based osculatory interpolant, in Section 3, we construct the more general structure of bivariate interpolation functions, and we discuss the special cases of the structure in Section 4. Also, error estimations (Section 5) for the interpolation are determined, and numerical examples are given in Section 6 to show the effectiveness of the results.

2 General Structure of Univariate Interpolant Function

Kahng[1] has employed the interpolation function

$$Q(x) = f_0(a_0 + g_0(x)f_1(a_1 + g_1(x)f_2(a_2 + \cdots + g_{n-1}(x)f_n(a_n)))), \tag{2.1}$$

to treat the univariate interpolation in a unified manner. This function can also be expressed as

$$Q(x) = f_0\{D_0(x)\},$$

where

$$\begin{aligned} D_i(x) &= a_i + g_j(x)f_{i+1}\{D_{i+1}(x)\}, \qquad i, j = 0, 1, \cdots, n-1, \\ D_n(x) &= a_n. \end{aligned}$$

In fact, it can be modified to the following scheme:

Given a set of real points

$$X_n = \{x_0, x_1, \cdots, x_n\} \subset [a, b] \subset \mathbf{R}$$

and a function $f(x)$ defined on $[a, b]$, consider the problem of interpolating a function $f(x)$ and its initial $m_i - 1$ derivatives of $f(x)$ at $X_n = \{x_0, x_1, \cdots, x_n\}$, namely,

$$f^{(k)}(x_i), \qquad i = 0, 1, \cdots, n;\ k = 0, 1, \cdots, m_i - 1.$$

Suppose that the set X_n is divided into $u + 1$ subsets X_n^s,

$$X_n^s = \{x_{c_s},\ x_{c_s+1},\ \cdots,\ x_{d_s}\}, \qquad s = 0, 1, \cdots, u.$$

The subsets may be achieved by reordering the interpolation points if necessary. Obviously, one can get

$$\sum_{s=0}^{u}(d_s - c_s + 1) = n + 1.$$

Then the function $Q(x)$ should be written as

$$Q(x) = f_0(a_0(x) + g_0(x)f_1(a_1(x) + g_1(x)f_2(a_2(x) + \cdots + g_{u-1}(x)f_u(a_u(x))))), \tag{2.2}$$