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Nonstandard Analysis Methods for Separations in [0, 1]-topological Spaces*

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Abstract: In nonstandard enlargement, the separations are characterized by nonstandard analysis methods in $[0, 1]$-topological spaces. Firstly, the monads of fuzzy point in $[0, 1]$-topological spaces are described with remote-neighborhoods in nonstandard enlarged model. Then the nonstandard characterizations of separations in $[0, 1]$-topological space are given by the monads. At last, relations of these separations are investigated.

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1 Introduction and Preliminary

Separations are very important and basic concepts in general topology (see [1]). They are different from other notions, such as connectivity and compactness, which come from the self-development of topology, not the generalization of real. In $[0, 1]$-topological spaces, fuzzy separations are more difficult. Fuzzy separations initiated by Chang[2] without "point" in 1968. After fuzzy points were defined in [3], Wang[4] rebuilt the fuzzy separations with fuzzy points and remote-neighborhoods.

In 1960s, Robinson[5] proposed nonstandard analysis methods, and then the methods were used in many mathematical branches (see [6–8]), especially in topology (see [9–10]). However, it was rarely used in fuzzy topology. In [11], the monads of three kinds of neighborhood structures were defined, and some properties of these monads were discussed. In [12], the nonstandard characterizations of Moore-Smith convergence, in $[0, 1]$-topological

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space, were shown. In the present, separations in $[0,1]$-topological spaces are investigated by nonstandard analysis methods.

In this paper, firstly, the monad of a fuzzy point in $[0,1]$-topological spaces is described with remote-neighborhoods in the nonstandard enlarged model. Then the nonstandard characterizations of the separations in $[0,1]$-topological space are shown by the monads. At last, the relations of these separations are investigated. It enriches the nonstandard analysis theory, and provides a new way to research $[0,1]$-topology.

Now, we give some notions and conclusions in nonstandard analysis and $[0,1]$-topology.

Let S be an infinite individual set. The superstructure $V(S)$ on S is the union

$$V(S) = \bigcup_{k\in\mathbf{N}_0} V_k(S) = V_0(S) \cup V_1(S) \cup V_2(S) \cup \cdots,$$

where $\mathbf{N}_0 = \{0\} \cup \mathbf{N}$, $\mathbf{N}$ denotes the set of natural numbers, and $V_k(S)$ are defined inductively by

$$V_0(S) = S, \qquad V_k(S) = V_{k-1}(S) \cup 2^{V_{k-1}(S)}, \qquad k = 1, 2, \cdots,$$

where 2^Y denotes the power set of Y.

Definition 1.1 *Let $V(S)$ be a superstructure on infinite individual set S. The superstructre $V(S)$ is called a nonstandard enlargement (or a nonstandard enlarged model) if there exists a mapping $* : V(S) \to V(S)$ such that the following conditions are satisfied:*

(1) ${}^*(\emptyset) = \emptyset$;

(2) $S \subset {}^*S$;

(3) *Transfer principle holds, i.e.,*

$$\models \alpha \quad \leftrightarrow \quad {}^*\!\models {}^*\alpha,$$

where α is a bounded quantifier formula in language $\mathcal{L}_{V(S)}$;

(4) *Concurrent principle holds, i.e., for any concurrent relation $r \in V(S)$, there is a $y \in V({}^*S)$ such that*

$$\langle {}^*x, y\rangle \in {}^*r, \qquad x \in \mathrm{dom}(r).$$

Throughtout this paper, let X be a classical set, $\mathbf{R}$ be the set of real numbers, $V({}^*S)$ be a nonstandard enlargement, and $X \cup \mathbf{R} \subseteq S$. Thus,

$$X \cup \mathbf{R} \subset {}^*(X \cup \mathbf{R}) = {}^*X \cup {}^*\mathbf{R}.$$

In fuzzy mathematics, a mapping $A : X \to [0,1]$ is called a fuzzy set, and $\mathcal{F}(X)$ denotes the collection of all fuzzy sets on X. Specially, a fuzzy set, which is only valued $\lambda(>0)$ at $x \in X$ and vanished at other points, is called a fuzzy point on X and denoted by x_λ. $B(X)$ denotes the collection of all fuzzy points on X.

By transfer principle, we have that a mapping ${}^*A : {}^*X \to {}^*[0,1]$ is a $*$-fuzzy set, ${}^*\mathcal{F}(X)$ and ${}^*B(X)$ denote the collection of all $*$-fuzzy sets and $*$-fuzzy points, respectively.

Definition 1.2 *A family $\delta \subseteq \mathcal{F}(X)$ is called a $[0,1]$-topology on X, and (X, δ) is said to be a $[0,1]$-topological space, if the following conditions are satisfied:*

(1) $0_X, 1_X \in \delta$, *where 0_X and 1_X denote the mapping $0_X : X \to \{0\}$ and $1_X : X \to \{1\}$, respectively;*