Communications
in
Mathematical
Research
28(4)(2012), 324–336

A New System of Generalized Variational Inclusions Involving H-η-monotone Operators in Uniformly Smooth Banach Spaces*

Feng Shi-qiang and Gao Da-peng
(*School of Mathematics and Information, China West Normal University, Nanchong, Sichuan, 637002*)

Communicated by Ji You-qing

Abstract: In this paper, we introduce and study a new system of generalized variational inclusions involving H-η-monotone operators in uniformly smooth Banach spaces. Using the resolvent operator technique associated with H-η-monotone operators, we prove the approximation solvability of solutions using an iterative algorithm. The results in this paper extend and improve some known results from the literature.
Key words: uniformly smooth Banach space, H-η-monotone operator, resolvent operator technique, system of generalized variational inclusion, iterative algorithm
2000 MR subject classification: 49J40, 47H10, 47H19
Document code: A
Article ID: 1674-5647(2012)04-0324-13

1 Introduction

Variational inequalities and variational inclusions are among the most interesting and important mathematical problems and have been studied intensively in the past years since they have wide applications in mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium, and engineering sciences, etc. (see, for example, [1–22]). Various kinds of iterative algorithms for solving the variational inequalities and inclusions have been developed by many authors. For details, we can refer to [1–20] and the references therein. Among these methods, the resolvent operator techniques for solving variational inequalities and variational inclusions are interesting and important.

On the other hand, monotonicity techniques were extended and applied in recent years because of their importance in the theory of variational inequalities, complementarity problems, and variational inclusions. In [1–3], Huang *et al.* introduced several kinds of mono-

*Received date: Feb. 15, 2011.
Foundation item: The NSF (60804065) of China and the Foundation (11A028) of China West Normal University.

tone operators, which gradually extended η-subdifferential operators, maximal η-monotone operators and H-monotone operators. They also studied some classes of varational inclusions using the resolvent operators associated with related monotone operators. Fang and Huang[4] further introduced a new class of generalized monotone operators, (H,η)-monotone operators, which provided a unifying framework for classes of maximal monotone operators, maximal η-monotone operators, and H-monotone operators. They also studied a class of variational inclusions using the resolvent operator associated with an (H,η)-monotone operator. Lou *et al.*[5] extended the concept of resolvent operators associated with (H,η)-monotone operators to new H-η-monotone operators. By using the new resolvent operator technique, they studied a system of variational inclusions involving H-η-monotone operators in Banach spaces. Motivated and inspired by the above works, in this paper, we explore the approximation solvability of a new system of generalized variational inclusion problems based on H-η-monotone resolvent operator techniques. Our results improve and extend the rencent ones.

2 Preliminaries

Let E be a real Banach space equipped with the norm $\|\cdot\|$. Denote by $\langle\ \cdot,\ \cdot\rangle$ the dual pair between E and its dual space E^*, by 2^E the family of all nonempty subsets of E, and by $CB(E)$ the family of all nonempty closed and bounded subsets of E. Let $D(\ \cdot,\ \cdot\ )$ be the Hausdorff metric on $CB(E)$ defined by

$$D(A,B)=\max\{\sup_{x\in A} d(x,B),\ \sup_{y\in B} d(A,y)\},\qquad A,B\in CB(E).$$

Let $J: E\longrightarrow 2^{E^*}$ be the normalized duality mapping defined by

$$J(u)=\{f\in E^*,\ \langle f,u\rangle=\|u\|^2,\ \|u\|=\|f\|_{E^*}\},\qquad u\in E.$$

It is well known that if E is smooth, then J is single valued, and if $E\equiv H$, a Hilbert space, then J is an identity mapping.

The following concepts and results are needed in the sequel.

Definition 2.1[6] *Let $P: E\longrightarrow E^*$, $g: E\longrightarrow E$, and $\eta: E\times E\longrightarrow E$ be single-valued mappings. Then*

(1) *P is said to be α-strongly η-monotone if there exists a constant $\alpha>0$ such that*

$$\langle P(u)-P(v),\ \eta(u,v)\rangle\geq\alpha\|u-v\|^2,\qquad u,v\in E;$$

(2) *g is said to be k-strongly accretive, if there exists a constant $k>0$ such that*

$$\langle g(u)-g(v),\ j(u-v)\rangle\geq k\|u-v\|^2$$

for any $u,v\in E$, $j(u-v)\in J(u-v)$, where j is a selection of set-valued mapping J;

(3) *η is said to be τ-Lipschitz continuous, if there exists a constant $\tau>0$ such that*

$$\|\eta(u,v)\|\leq\tau\|u-v\|,\qquad u,v\in E.$$

Definition 2.2[4] *Let $T: E\longrightarrow E^*$ be a single-valued operator. Then the operator T is said to be*