COMMUNICATIONS
IN
MATHEMATICAL
RESEARCH
29(1)(2013), 23–31

Superlinear Fourth-order Elliptic Problem without Ambrosetti and Rabinowitz Growth Condition*

WEI YUAN-HONG[1], CHANG XIAO-JUN[2,3] AND LÜ YUE[2]

(*1. Institute of Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, 100080*)

(*2. College of Mathematics, Jilin University, Changchun, 130012*)

(*3. Chern Institute of Mathematics, Nankai University, Tianjin, 300071*)

Communicated by Li Yong

Abstract: This paper deals with superlinear fourth-order elliptic problem under Navier boundary condition. By using the mountain pass theorem and suitable truncation, a multiplicity result is established for all $\lambda > 0$ and some previous result is extended.

Key words: fourth-order elliptic problem, variational method, mountain pass theorem, Navier boundary condition

2000 MR subject classification: 35J30, 35J35

Document code: A

Article ID: 1674-5647(2013)01-0023-09

1 Introduction and Main Results

Fourth-order elliptic problems are usually used to describe some phenomena appeared in different physical, engineering and other sciences. Lazer and McKenna[1] studied the problem of nonlinear oscillation in a suspension bridge and they presented a mathematical model for the bridge that took account of the fact that the coupling provided by the stays connecting the suspension cable to the deck of the road bed is basically nonlinear. Also, Liu and Feng[2] pointed out that this kind of problem furnishes a good model to the static deflection of an elastic plate in a fluid. Ahmed and Harbi[3] indicated that this problem also arises in such as communication satellites, space shuttles, and space stations, which are equipped with large antennas mounted on long flexible masts (beams). Fourth-order elliptic problems have been studied extensively in recent years, and we refer the reader to [4–9] and the references

*Received date: March 15, 2010.
Foundation item: The 985 Program of Jilin University and the Science Research Foundation for Excellent Young Teachers of College of Mathematics at Jilin University.

therein.

Consider the following fourth-order elliptic problem:

$$\begin{cases} \Delta^2 u + c\Delta u = \lambda f(x, u) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Δ^2 is the biharmonic operator, c is a constant, $\Omega \subset \mathbf{R}^N$ is a bounded smooth domain and $f(x, s)$ is a continuous function on $\bar{\Omega} \times \mathbf{R}$.

Denote

$$F(x, s) = \int_0^s f(x, t)\mathrm{d}t,$$
$$H(x, s) = sf(x, s) - 2F(x, s).$$

We assume that $f(x, s)$ satisfies the following hypotheses:

(H1) $\lim\limits_{s\to 0} \dfrac{f(x, s)}{s} = 0$ uniformly for a.e. $x \in \Omega$;

(H2) There exist positive constants C_1 and C_2 such that

$$|f(x, s)| \leqslant C_1 + C_2|s|^p,$$
$$1 \leqslant p < q = \begin{cases} \dfrac{N+2}{N-2}, & N \geqslant 3, \\ +\infty, & N \leqslant 2; \end{cases}$$

(H3) $\lim\limits_{|s|\to +\infty} \dfrac{F(x, s)}{s^2} = +\infty$ uniformly for a.e. $x \in \Omega$;

(H4) There exists a $C_* > 0$ such that

$$H(x, t) \leqslant H(x, s) + C_*$$

for all $0 < t < s$ or $s < t < 0$, $x \in \Omega$.

To obtain nontrivial solutions of the problem (1.1) by applying variational method, one often uses the Ambrosetti-Rabinowitz condition (see [10]), i.e.,

(AR) There are constants $\theta > 0$ and $s_0 > 0$ such that

$$0 < (2 + \theta)F(x, s) \leqslant f(x, s)s, \quad |s| > s_0, \qquad x \in \Omega.$$

This condition ensures the compactness of the corresponding functional, however, it eliminates many nonlinearities. To avoid the condition (AR), many approaches were developed. Costa and Magalhães[11] studied the problem (1.1) via replacing the condition (AR) by

$$\liminf_{s\to\infty} \frac{sf(x, s) - 2F(x, s)}{|s|^\mu} \geqslant k > 0 \qquad \text{uniformly for a.e. } x \in \Omega,$$

where $\mu \geqslant \mu_0 > 0$. Willem and Zou[12] assumed that $H(x, s)$ is increasing in s and

$$sf(x, s) \geqslant 0, \quad s \in \mathbf{R}; \quad sf(x, s) \geqslant C_0|s|^\mu, \quad |s| \geqslant s_0 > 0, \qquad x \in \Omega,$$

where $\mu > 2$ and $C_0 > 0$, in place of the condition (AR). Recently, by using the assumptions (H1)–(H4), Miyagaki and Souto[13] obtained a nontrivial weak solution in the case of second-order elliptic problem.

For the fourth-order problem (1.1), Zhang and Li[14] obtained at least two nontrivial solutions by means of Morse theory and local linking when f is sublinear at infinity. By using the linking theorem, Qian and Li[15] obtained one nontrivial solution if f is superlinear and satisfies the Ambrosetti-Rabinowitz condition, and two nontrivial solutions if f is asymptotically linear as s is large enough. An and Liu[2] also established the existence of at least