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Strong Consistency of M Estimator in Linear Model for $\tilde{\varphi}$-mixing Samples*

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Abstract: The strong consistency of M estimator of regression parameter in linear model for $\tilde{\varphi}$-mixing samples is discussed by using the classic Rosenthal type inequality. We get the strong consistency of M estimator under lower moment condition, which generalizes and improves the corresponding ones for independent sequences.

Key words: $\tilde{\varphi}$-mixing sample, M estimator, strong consistency

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1 Introduction

Consider the following linear model:

$$y_i = \boldsymbol{x}_i'\boldsymbol{\beta}_0 + e_i, \qquad i = 1, 2, \cdots, n, \ n \in \mathbf{N}, \tag{1.1}$$

where $\boldsymbol{x}_i$ $(i = 1, 2, \cdots, n)$ is a known p-dimensional vector, $\boldsymbol{\beta}_0$ is an unknown p-dimensional regression parametric vector, and $e_1, e_2, \cdots, e_n$ are random errors. Let f be a convex function on $\mathbf{R}$. The M estimator of $\boldsymbol{\beta}_0$ is $\hat{\boldsymbol{\beta}}_n$ satisfying the following equation:

$$\sum_{i=1}^{n} f(y_i - \boldsymbol{x}_i'\hat{\boldsymbol{\beta}}_n) = \min_{\boldsymbol{\beta}\in\mathbf{R}^p} \sum_{i=1}^{n} f(y_i - \boldsymbol{x}_i'\boldsymbol{\beta}). \tag{1.2}$$

The scope of M estimator is very wide containing the least squares estimator, maximum likelihood estimator *etc.* Since Huber[1] studied the M estimator of regression parameter in linear model, many authors have shown great interest in this field and obtained many useful results; see, for example, [2–5].

Throughout this paper, we use the following notations: Let

$$\boldsymbol{\alpha} = (a_1, \ a_2, \ \cdots, \ a_p)'$$

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be a p-dimensional vector, and

$$\|\boldsymbol{\alpha}\|^2 \doteq \sum_{i=1}^{p} a_i^2 = \boldsymbol{\alpha}'\boldsymbol{\alpha}, \qquad |\boldsymbol{\alpha}| \doteq \max_{1\le i\le p} |a_i|.$$

Denote

$$\boldsymbol{S}_n = \sum_{i=1}^{n} \boldsymbol{x}_i \boldsymbol{x}_i'.$$

Assume that $\boldsymbol{S}_n^{-1}$ exists, and

$$d_n \doteq \max_{1\le i\le n} \boldsymbol{x}_i' \boldsymbol{S}_n^{-1} \boldsymbol{x}_i.$$

Let f be a convex function, ψ_- and ψ_+ denote the left derivative and right derivative of function f, respectively. $a_n = O(b_n)$ denotes that there exists a positive constant C such that $\left|\dfrac{a_n}{b_n}\right| \le C$ for all sufficiently large n. C and C_i $(i \ge 1)$ are positive constants which may be different in various places.

As for the sufficient condition for the strong consistency of M estimator, Chen and Zhao[2] obtained the following result:

Theorem 1.1[2] *Let* $e_1, e_2, \cdots, e_n, \cdots$ *be a sequence of independent random variables with identical distribution, and* f *be a convex function satisfying the following two conditions*:

(1) *There exist constants* $l_1 > 0$ *and* $l_2 > 0$ *such that*

$$E(f(e_1 + u) - f(e_1)) \ge l_1 u^2, \qquad |u| < l_2;$$

(2) *There exists a constant* $\Delta > 0$ *such that*

$$E|\psi_+(e_1 \pm \Delta)|^m \le h_m < \infty, \qquad m = 1, 2, \cdots$$

If there exists a δ *with* $0 < \delta \le 1$ *such that* $d_n = O(n^{-\delta})$, *then,* $\hat{\boldsymbol{\beta}}_n$ *is the strong consistency estimator of* $\boldsymbol{\beta}_0$.

Yang[3] improved the result of Theorem 1.1 and obtained the following Theorem 1.2 and Theorem 1.3.

Theorem 1.2[3] *Let* $e_1, e_2, \cdots, e_n, \cdots$ *be a sequnce of independent random variables with identical distribution, and* f *be a convex function satisfying the following two conditions*:

(1) *There exist constants* $l_1 > 0$ *and* $l_2 > 0$ *such that*

$$E(f(e_1 + u) - f(e_1)) \ge l_1 u^2, \qquad |u| < l_2;$$

(2) *There exist constants* $h_0 > 0$, $\Delta > 0$ *and* $0 < \delta \le 1$ *such that*

$$E|\psi_+(e_1 \pm \Delta)|^{2/\delta} \le h_0 < \infty$$

and

$$d_n = O(n^{-\delta}).$$

Then, $\hat{\boldsymbol{\beta}}_n$ *is the strong consistency estimator of* $\boldsymbol{\beta}_0$.

Theorem 1.3[3] *Let* $e_1, e_2, \cdots, e_n, \cdots$ *be a sequence of independent random variables, and* f *be a convex function satisfying the following two conditions*: