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On the Expected Present Value of Total Dividends in a Risk Model with Potentially Delayed Claims

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Abstract: In this paper, we consider a risk model in which two types of individual claims, main claims and by-claims, are defined. Every by-claim is induced by the main claim randomly and may be delayed for one time period with a certain probability. The dividend policy that certain amount of dividends will be paid as long as the surplus is greater than a constant dividend barrier is also introduced into this delayed claims risk model. By means of the probability generating functions, formulae for the expected present value of total dividend payments prior to ruin are obtained for discrete-type individual claims. Explicit expressions for the corresponding results are derived for K_n claim amount distributions. Numerical illustrations are also given.

Key words: compound binomial model, delayed claim, dividend, expected present value

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1 Introduction

The barrier strategy was initially proposed by De Finetti[1] for a binomial risk model. More general barrier strategies for a risk process have been studied in a number of papers; for example, see [2–7] and references therein. The main focus is on optimal dividend payouts and problems associated with time of ruin, under various barrier strategies and other economic conditions. In fact, the study on risk models with dividend payments has been one of the major interests in the risk theory literature.

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In reality, insurance claims may be delayed due to various reasons. Since the work by Waters and Papatriandafylou[8], risk models with this special feature have been discussed by many authors in the literature. For example, Yuen and Guo[9] studied a compound binomial model with delayed claims and obtained recursive formulas for the finite time ruin probabilities. Xiao and Guo[10] also studied this risk model and derived an upper bound for the ruin probabilities. Xie and Zou[11] studied the ultimate ruin probability for a risk model with delayed claims, and exact analytical expressions for the ruin functions were obtained. Yuen *et al.*[12] studied a risk model with delayed claims, in which the time of delay for the occurrence of a by-claim is assumed to be exponentially distributed. A framework of delayed claims is built by introducing two kinds of individual claims, namely, main claims and by-claims, allowing possible delays of the occurrences of by-claims. All risk models described above relied on the assumption that each main claim induces a by-claim to a certainty.

Motivated by these works, in this paper, we consider a compound binomial risk model with delayed claims and dividend payments that are ruled by a constant barrier. In our model, every by-claim is induced by the associated main claim randomly. It is obvious that the incorporation of the randomness of delayed claim and dividend payments makes the problem more interesting. It also complicates the analysis of the risk model. We use the technique of generating functions to calculate the expected present value of total dividends for this risk model. The paper is structured as follows: In Section 2, we define the model of interest, describes various payments, including the premiums, claims and dividends, and lists the notations. In Section 3, differential equations with certain boundary conditions are developed for the expected present value of total dividend payments prior to ruin. Then a general formula which allows us to evaluate the expected present value of the total discounted dividends is derived by using the technique of generating functions. Moreover, closed-form solutions for the expected present value of dividends are obtained for K_n claim size distributions in Section 4. Numerical examples are also provided to illustrate the applicabilities of our main results and the effects of the delay of claims on the expected present value of dividends in Section 4.

2 Model Description and Notations

Here, we consider a discrete time model which involves two types of insurance claims, i.e., the main claims and the by-claims. Denote the discrete time units by $k = 0,\ 1,\ 2,\ \cdots$ In any time period, the probability of having a main claim is q, $0 < q < 1$. The occurrences of main claims in different time periods are independent. It is assumed that each main claim induces a by-claim with probability φ, $0 < \varphi < 1$, and the main claim does not induce a by-claim with probability $1 - \varphi$. Moreover, if the main claim induces a by-claim, then the by-claim and its associated main claim may occur simultaneously with probability θ, or the occurrence of the by-claim may be delayed to the next time period with probability $1 - \theta$. All claim amounts are independent, positive and integer valued. The main claim amounts $X_1,\ X_2,\ \cdots$ are independent and identically distributed (i.i.d.). Put $X = X_1$. Then the