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Invariants for Automorphisms of the Underlying Algebras Relative to Lie Algebras of Cartan Type

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Abstract: Let X denote a finite or infinite dimensional Lie algebra of Cartan type W, S, H or K over a field of characteristic $p \geq 3$. In this paper it is proved that certain filtrations of the underlying algebras are invariant under the admissible groups relative to Lie algebras of Cartan type X.

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1 Introduction

Premet and Strade[1] finished the complete classification of finite dimensional simple Lie algebras over an algebraically closed field of characteristic $p > 3$. We know that Lie algebras of Cartan type play an important role in the modular Lie algebra theory. In [2–6], the automorphism groups of Lie algebras of Cartan type were sufficiently studied. In this paper we obtain the invariance of certain filtration structures of the underlying algebras of finite or infinite dimensional Lie algebras of Cartan type. The results are used to establish a correspondence between the admissible automorphism groups of the underlying algebras $\mathcal{O}$ relative to X and the automorphism group of X, and an isomorphism from the admissible homogeneous automorphism group of $\mathcal{O}$ onto the homogeneous automorphism group of X, where X stands for a finite or an infinite dimensional Lie algebra of Cartantype W, S, H or K.

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2 Beginning

In this paper $\mathbb{F}$ denotes a field of characteristic $p \geq 3$. $\mathbf{Z}_+$ and $\mathbf{N}$ denote the positive integer set and nonnegative integer set, respectively. Fix an integer $m > 2$ and the index set $I := \{1, 2, \cdots, m\}$. Denote by $\mathbf{N}^m$ the additive monoid of m-tuples of nonnegative integers. Suppose $m = 2r$ or $2r + 1$, where $r \in \mathbf{Z}_+$. Set

$$i' := \begin{cases} i + r, & 1 \leq i \leq r; \\ i - r, & r < i \leq 2r, \end{cases} \qquad \tau(i) := \begin{cases} 1, & 1 \leq i \leq r; \\ -1, & r < i \leq 2r. \end{cases}$$

We recall the necessary definitions and facts. Denote by $\mathcal{O}(m)$ the divided power algebra with $\mathbb{F}$-basis $\{x^\alpha \mid \alpha \in \mathbf{N}^m\}$. For $i \in I$, let ∂_i be the special derivation of $\mathcal{O}(m)$ such that $\partial_i(x^\alpha) = x^{\alpha-\varepsilon_i}$ for all $\alpha = (\alpha_1, \cdots, \alpha_m) \in \mathbf{N}^m$ (see [4, 7]). Hereafter, ε_i is the m-tuple with 1 at the i-position and 0 elsewhere. For simplicity, write x_i for x^{ε_i}. Notice that

$$W(m) := \Big\{ \sum_{i=1}^{m} a_i\partial_i \mid a_i \in \mathcal{O}(m) \Big\}$$

is a Lie-subalgebra of $\operatorname{Der} \mathcal{O}(m)$. Set

$$S(m) := \operatorname{span}_{\mathbb{F}}\{\mathcal{D}_{ij}(a) \mid a \in \mathcal{O}(m),\ i, j \in I\},$$

where $\mathcal{D}_{ij}(a) := \partial_j(a)\partial_i - \partial_i(a)\partial_j$;

$$H(m) := \{\mathcal{D}_H(a) \mid a \in \mathcal{O}(m)\},$$

where $m = 2r$ is even,

$$\mathcal{D}_H(a) := \sum_{i=1}^{m} \tau(i)\partial_i(a)\partial_{i'};$$

and

$$K(m) := \{\mathcal{D}_k(a) \mid a \in \mathcal{O}(m)\},$$

where $m = 2r + 1$ is odd and

$$\mathcal{D}_k(a) := \Big(2a - \sum_{i=1}^{m-1} x_i\partial_i(a)\Big)\partial_m + \sum_{i=1}^{m-1} (x_{i'}\partial_m(a) + \tau(i)\partial_i(a))\partial_{i'}.$$

Then $X(m)$ is an infinite-dimensional subalgebra of $W(m)$, where $X = W$, S, H or K. Let $\underline{n} \in \mathbf{Z}_+^m$ be an m-tuple of positive integers. Then

$$\mathcal{O}(m;\, \underline{n}) := \operatorname{span}_{\mathbb{F}}\{x^\alpha \mid \alpha \in \mathbf{N},\, \alpha_i < p^{n_i} \text{ for each } i \in I\}$$

is a subalgebra of $\mathcal{O}(m)$. Define

$$W(m;\, \underline{n}) := \sum_{i \in I} \mathcal{O}(m;\, \underline{n})\partial_i.$$

Then $W(m;\, \underline{n})$ is a finite dimensional subalgebra of $W(m)$. Put

$$X(m;\, \underline{n}) := X(m) \cap W(m;\, \underline{n}), \qquad X = W,\, S,\, H \text{ or } K.$$

Then, the second derived subalgebra $X(m;\, \underline{n})^{(2)}$ is a simple Lie algebra (see [4, 6–7]). In the sequel, $X(m;\, \underline{n})$ is identified with a subalgebra of $\operatorname{Der}\mathcal{O}(m;\, \underline{n})$. For the sake of simplicity, in the following we often use X to stand for the Lie algebras $X(m)$ or $X(m;\, \underline{n})^{(2)}$, where $X = W$, S, H or K. The underlying associative algebra of X is denoted by $\mathcal{O}_X$, or $\mathcal{O}$, in brief. Note that the underlying algebra of $X(m;\, \underline{n})^{(2)}$ is $\mathcal{O}(m;\, \underline{n})$.