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Stochastic Nonlinear Beam Equations with Lévy Jump

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Abstract: In this paper, we study stochastic nonlinear beam equations with Lévy jump, and use Lyapunov functions to prove existence of global mild solutions and asymptotic stability of the zero solution.

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1 Introduction

Consider a wide class of abstract stochastic beam equations with Lévy jump in a separable Hilbert space H:

$$\begin{cases} u_{tt} + A^2 u + g(u, u_t) + m(\|B^{\frac{1}{2}} u\|^2) Bu = \sigma(u, u_t)\dot{W} + \displaystyle\int_Z f(u, u_t, z)\tilde{N}(\mathrm{d}z, t), \\ u(0) = u_0, \qquad u_t(0) = u_1, \end{cases} \tag{1.1}$$

where A and B are positive self-adjoint operators, m is a nonnegative function in $C^1([0, +\infty))$, W is a Wiener process, and $\tilde{N}$ is the compensated Poisson measure.

In [1], a model for the transversal deflection of an extensible beam of a natural length l was proposed as follows:

$$\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial^4 u}{\partial x^4} = \left[a + b\int_0^l \left(\frac{\partial u}{\partial x}\right)^2 \mathrm{d}x\right] \frac{\partial^2 u}{\partial x^2}. \tag{1.2}$$

Chow and Menaldi[2] considered a stochastic extensible beam equation which describes large amplitude vibrations of an elastic panel excited by aerodynamic forces. They studied

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a stochastic beam equation as follows:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2} - \Big[a + b\int_0^l \Big(\frac{\partial u}{\partial x}\Big)^2 \mathrm{d}x\Big]\frac{\partial^2 u}{\partial x^2} + \gamma\frac{\partial^4 u}{\partial x^4} \\
&= g\Big(t,\ x,\ \frac{\partial u}{\partial t},\ \frac{\partial u}{\partial x}\Big) + \sum_{k=1}^{\infty}\sigma_k\Big(t,\ x,\ \frac{\partial u}{\partial t},\ \frac{\partial u}{\partial x}\Big)\dot{\omega}_k, \qquad x \in [0,\ l],\ t \geq 0, \\
&u(t,\ 0) = u(t,\ l) = \frac{\partial u}{\partial x}(t,\ 0) = \frac{\partial u}{\partial x}(t,\ l) = 0, \qquad t \geq 0.
\end{aligned} \tag{1.3}$$

Brzeźniak *et al.*[3] studied (1.3), gave a wide class of abstract stochastic beam equations, proved nonexplosion of mild solutions to (1.3), and established the asymptotic stability of the zero solution when the damping term g is of the form

$$g(u,\ u_t) = \beta u_t, \qquad \beta \geq 0.$$

Recently, the stochastic partial differential equations driven by jump processes have been studied by some scholars. Albeverio *et al.*[4] studied the stochastic reaction diffusion equations driven by Poisson random measures, and established the existence and uniqueness of the solution under growth and Lipschitz conditions. Fournier[5] used Malliavin calculus to study the continuity of the law of the weak solution of the stochastic reaction diffusion equations driven by Poisson random measures. Mueller[6] constructed a minimal solution for the stochastic heat equation driven by non-negative Lévy noise with coefficients of polynomial growth. Mytnik[7] established a weak solution for the stochastic partial equation driven by a one sided, α-stable noise without negative jumps. Röckner and Zhang[8] studied the stochastic evolution equations driven by both Brownian motion and Poisson point processes, and obtained the existence and uniqueness results of the equations.

The purpose of the present paper is to deal with the stochastic beam equations driven by white noise and Poisson noise. The model describes large amplitude vibrations and rare events with low frequency and sudden occurrence vibrations of an elastic panel excited by aerodynamic forces. We prove the existence of global solutions of the system (1.1), by using the technique of constructing a proper Lyapunov function (see [9]), i.e., we assume that there exists a Lyapunov function $V(u) : \mathcal{H} \to \mathbf{R}$ of the system (1.1) such that

$$V_R = \inf_{\|u\| \geq R} V(u) \to \infty \quad \text{as } R \to \infty, \qquad \frac{\mathrm{d}V}{\mathrm{d}t} < c_1 V,$$

and the coefficient of system (1.1) satisfies some conditions, then there exists a global solution of the system (1.1) for all $t \geq t_0$.

The stability result in this paper depends on the form of $g(u, u_t)$. We can establish the asymptotic stability of zero solution when the damping term g is of the form

$$g(u, u_t) = \beta u_t + h(|u|^2)u, \qquad \beta \geq 0,$$

where $h \in C^1([0, \infty))$ is a nonnegative function.

2 Notations and Preliminaries

Let H be a separable Hilbert space with the norm and the inner product denoted by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$, respectively. Suppose that