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Generalized Extended tanh-function Method for Traveling Wave Solutions of Nonlinear Physical Equations

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Abstract: In this paper, the generalized extended tanh-function method is used for constructing the traveling wave solutions of nonlinear evolution equations. We choose Fisher's equation, the nonlinear schrödinger equation to illustrate the validity and advantages of the method. Many new and more general traveling wave solutions are obtained. Furthermore, this method can also be applied to other nonlinear equations in physics.

Key words: generalized tanh-function method, nonlinear Schrödinger equation, Fisher's equation

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1 Introduction

It is well known that the nonlinear phenomena is very important in variety of the scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, capillary-gravity waves and chemical physics. Most of these phenomena are described by the nonlinear partial differential equations. So exact solutions of the nonlinear partial differential equations play an essential role in the nonlinear science. For this end, various methods,

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such as the inverse scattering method (see [1]), the Hirota's bilinear technique (see [2]), and truncated Painlv́e expansion (see [3]) have been developed to obtain exact solutions. The tanh method presented by Malfliet[4–6] is a powerful solution method to get the exact traveling wave solutions. Later, Fan *et al.*[7–8] proposed an extended tanh-function method and obtained the new traveling wave solutions which cannot be obtained by tanh-function method. Recently, El-Wakil and Abdou[9] modified the extended tanh-function method and obtained some new exact solutions. In this paper, we extended the modified tanh-function method to get the new exact traveling wave solutions. For illustration, we apply this method to Fisher's equation and the nonlinear Schrödinger equation with general nonlinearity.

2 The Generalized Extend tanh-function Method

In this section, we give a brief description of the generalized extended tanh method. Consider the following nonlinear partial differential equation (PDE):

$$F(u,\ u_t,\ u_x,\ u_{tt},\ u_{xt},\ u_{uu},\ \cdots) = 0, \tag{2.1}$$

where $u = u(t,\, x)$ is an unknown function, F is a polynomial in $u = u(t,\, x)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

We first consider the traveling wave solutions of (2.1)

$$u(t,\, x) = U(\xi), \qquad \xi = \lambda(x - Vt),$$

and reduce (2.1) into the following ordinary differential equation (ODE):

$$F(U,\, -\lambda V U',\, \lambda U',\, V^2 U'',\, -\lambda V U'',\, \lambda^2 U'',\, \cdots) = 0, \tag{2.2}$$

where $U' = \dfrac{\mathrm{d}U}{\mathrm{d}\xi}$. The solutions can be expressed as the polynomial form

$$U(\xi) = S(Y(\xi)) = \sum_{k=0}^{M} a_k Y^k, \tag{2.3}$$

where the positive integer M can be determined by balancing the highest order derivative term with the nonlinear terms in (2.2), and Y is the solution of the Riccati equation

$$Y' = Y^2 + \alpha Y + b, \tag{2.4}$$

where α and b are constants to be determined. Substituting (2.3) and (2.4) into (2.2) and equating the coefficients of all powers Y^k to zero yield a system of algebraic equations for V, λ, a_0, a_i $(i = 1,\, 2,\, \cdots)$, from which the constants are obtained explicitly.

The Riccati equation (2.4) has general solutions as follows:

(I) If $\alpha = 0$ and $b = -1$, then

$$Y = -\tanh(-\xi) \quad \text{or} \quad -\coth(-\xi). \tag{2.5}$$

This method is the traditional tanh method (see [4–6]).

(II) If $\alpha = 0$ and b is an arbitrary constant, then

$$Y = \begin{cases} -\sqrt{-b}\tanh(-\sqrt{-b}\xi) \quad \text{or} \quad -\sqrt{-b}\coth(-\sqrt{-b}\xi), & b < 0; \\ -\dfrac{1}{\xi}, & b = 0; \\ \sqrt{b}\tan(\sqrt{b}\xi) \quad \text{or} \quad \sqrt{b}\cot(\sqrt{b}\xi), & b > 0. \end{cases} \tag{2.6}$$