COMMUNICATIONS
IN
MATHEMATICAL
RESEARCH
30(3)(2014), 207–221

Bifurcation in a Class of Planar Piecewise Smooth Systems with 3-parameters

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Communicated by Ma Fu-ming

Abstract: This paper is concerned with the bifurcation properties on the line of discontinuity of planar piecewise smooth systems. The existence of equilibria and periodic solutions with sliding motion in a class of planar piecewise smooth systems with 3-parameters is investigated in this paper using the theory of differential inclusion and tools of Poincaré maps.

Key words: piecewise smooth system, line of discontinuity, equilibria, periodic solution with sliding motion, bifurcation

2010 MR subject classification: 34A36, 34A60, 34C23, 37G10

Document code: A

Article ID: 1674-5647(2014)03-0207-15

DOI: 10.13447/j.1674-5647.2014.03.03

1 Introduction

Many dynamical systems that occur naturally in the description of physical processes are presented as piecewise smooth equations. For example, they arise in the case of impacts, they occur in mechanical systems if the effects of dry friction are considered, they frequently appear in control theory when discontinuous or impulse controls are involved (see [1–3]).

Piecewise smooth dynamical systems have been shown to exhibit many bifurcation phenomena that cannot be explained in terms of classical bifurcation theory for smooth systems. Examples include the bifurcation of equilibria of a planar piecewise smooth system when the discontinuity and the equilibria interact on each other (i.e., equilibria lying on the discontinuity boundary of phase space), grazing bifurcation and sliding phenomenon etc. Bifurcation theories of smooth systems are well understood and described in many textbooks (see [4–6]), whereas bifurcations of systems with discontinuous vector fields (or Filippov systems) are still the object of active research. In [7], a procedure to find all limit sets near bifurcating

Received date: Sept. 15, 2011.
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equilibria was presented in a class of hybrid systems. Leine[8] showed a variety of bifurcation phenomena of equilibria which can be observed in non-smooth continuous systems. Giannakopoulos and Pliete[9] investigated the bifurcation of equilibrium points and periodic trajectories of $\mathbb{Z}_2$-symmetry planar piecewise linear differential equations. Zang *et al.*[10] investigated the bifurcation properties of stationary points of a class of planar piecewise smooth systems with 3-parameters.

The focus of this paper is to investigate the bifurcation properties of stationary points on discontinuity and periodic solutions with sliding motion of a class of planar piecewise linear Filippov system with 3-parameters of the following form

$$\dot{u} = \begin{cases} \boldsymbol{A}_+(\boldsymbol{u} - \lambda_1 \boldsymbol{e}_2 - \lambda_2 \boldsymbol{e}_1), & \boldsymbol{u} \in \Omega_+, \\ \boldsymbol{A}_-(\boldsymbol{u} + \lambda_1 \boldsymbol{e}_2 + \lambda_3 \boldsymbol{e}_1), & \boldsymbol{u} \in \Omega_-, \end{cases} \tag{1.1}$$

where $\boldsymbol{u} = (x, y)^{\mathrm{T}} \in \mathbf{R}^2$, $\boldsymbol{A}_\pm = \begin{pmatrix} \alpha_\pm & \beta_\pm \\ -\beta_\pm & \alpha_\pm \end{pmatrix}$ are 2×2 real matrices, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the natural basis of $\mathbf{R}^2$, $\Omega_\pm = \big\{(x, y)^{\mathrm{T}} \in \mathbf{R}^2 : \pm x > 0\big\}$, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3) \in \mathbf{R}^3$ are parameters. Obviously, the set of points of discontinuity of the system (1.1) is

$$\Sigma = \{(x, y)^{\mathrm{T}} \in \mathbf{R}^2 : x = 0\}.$$

In [10], the bifurcation properties of stationary points of the system (1.1) were well investigated under the restriction $\alpha_+\beta_- - \alpha_-\beta_+ = 0$. In this paper, we remove this restriction and investigate the existence theorems of stationary points on the line of discontinuity and periodic solutions with sliding motion.

This paper is organized as follows: In Section 2, we introduce and discuss some basic assumptions and definitions. In Section 3, we study the existence of equilibria on the line of discontinuity. In Section 4, we investigate the existence of periodic solutions with sliding motion by means of Poincaré maps.

2 Basic Assumptions

In this section we introduce some basic assumptions for this work and discuss some consequences of our assumptions. We assume that

(H1) $4\det(\boldsymbol{A}_\pm) > (\mathrm{tr}(\boldsymbol{A}_\pm))^2 \neq 0$;

(H2) $\beta_\pm > 0$;

(H3) $\alpha_+\beta_- - \alpha_-\beta_+ > 0$.

Assumption (H1) implies that the matrices $\boldsymbol{A}_+$ and $\boldsymbol{A}_-$ possess a pair of complex eigenvalues $a_+ \pm \mathrm{i}b_+$ and $a_- \pm \mathrm{i}b_-$ with $a_\pm \neq 0$, respectively, that is to say the matrices $\boldsymbol{A}_\pm$ are invertible. Assumption (H2) assures that the flow of the system (1.1) develops surrounding the origin clockwise. From assumption (H3) it follows that the vector field on the line of discontinuity Σ is quadratic rather than linear.

Define

$$f^+ = \boldsymbol{A}_+(\boldsymbol{u} - \lambda_1 \boldsymbol{e}_2 - \lambda_2 \boldsymbol{e}_1), \qquad f^- = \boldsymbol{A}_-(\boldsymbol{u} + \lambda_1 \boldsymbol{e}_2 + \lambda_3 \boldsymbol{e}_1).$$