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Vertex-distinguishing IE-total Colorings of Cycles and Wheels

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Abstract: Let G be a simple graph. An IE-total coloring f of G refers to a coloring of the vertices and edges of G so that no two adjacent vertices receive the same color. Let $C(u)$ be the set of colors of vertex u and edges incident to u under f. For an IE-total coloring f of G using k colors, if $C(u) \neq C(v)$ for any two different vertices u and v of $V(G)$, then f is called a k-vertex-distinguishing IE-total-coloring of G, or a k-VDIET coloring of G for short. The minimum number of colors required for a VDIET coloring of G is denoted by $\chi_{vt}^{ie}(G)$, and is called the VDIET chromatic number of G. We get the VDIET chromatic numbers of cycles and wheels, and propose related conjectures in this paper.

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1 Introduction and Preliminaries

For an edge coloring (proper or not) of a graph G and a vertex v of G, denote by $S(v)$ the set of colors used to color the edges incident to v.

A proper edge coloring of a graph G is said to be vertex-distinguishing if each pair of vertices is incident to a different set of colors. In other words, $S(u) \neq S(v)$ whenever $u \neq v$. A graph G has a vertex-distinguishing proper edge coloring if and only if it has no more than one isolated vertex and no isolated edges. Such a graph is referred to as a *vdec*-graph. The minimum number of colors required for a vertex-distinguishing proper edge coloring

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of a *vdec*-graph G is denoted by $\chi'_s(G)$. The concept of vertex-distinguishing proper edge coloring has been considered in several papers (see [1–7]).

A general edge coloring (not necessarily proper) of a graph G is called vertex-distinguishing if $S(u) \neq S(v)$ is required for any two distinct vertices u and v. The point-distinguishing chromatic index of a *vdec*-graph G, denoted by $\chi_0(G)$, refers to the minimum number of colors required for a vertex-distinguishing general edge coloring of G. This parameter was introduced by Harary and Plantholt in [8]. In spite of the fact that the structure of complete bipartite graph is simple, it seems that the problem of determining $\chi_0(K_{m,n})$ is not easy, especially in the case $m = n$, as documented by papers of Horňák and Soták[9–10], Zagaglia Salvi[11–12] and Horňák and Zagaglia Salvi[13].

For a total coloring (proper or not) f of G and a vertex v of G, denote by $C_f(v)$, or simply $C(v)$ if no confusion arises, the set of colors used to color the vertex v as well as the edges incident to v. Let $\bar{C}(v)$ be the complementary set of $C(v)$ in the set of all colors we used. Obviously $|C(v)| \leq d_G(v) + 1$ and the equality holds if the total coloring is proper.

For a proper total coloring, if $C(u) \neq C(v)$, i.e., $\bar{C}(u) \neq \bar{C}(v)$ for any two distinct vertices u and v, then the coloring is called vertex-distinguishing (proper) total coloring and the minimum number of colors required for a vertex-distinguishing (proper) total coloring is denoted by $\chi_{vt}(G)$. This concept has considered in [14–15]. In [15], the authors give the following conjecture.

Conjecture 1.1 *Suppose that G is a simple graph and n_d is the number of the vertices of degree d, with $\delta \leq d \leq \Delta$. If k is the minimum positive integer such that $\binom{k}{d+1} \geq n_d$ for all d, with $\delta \leq d \leq \Delta$, then $\chi_{vt}(G) = k$ or $k + 1$.*

From [15] we know that the above conjecture is valid for complete graphs, complete bipartite graphs, path and cycle, etc.

In this paper we propose a kind of vertex-distinguishing general total coloring. The relationship of this coloring and vertex-distinguishing proper total coloring is similar to the relationship of vertex-distinguishing general edge coloring and vertex-distinguishing proper edge coloring.

When we define a proper total coloring of a graph G, we need three conditions for a total coloring which are listed as follows:

Condition (v): No two adjacent vertices receive the same color;

Condition (e): No two adjacent edges receive the same color;

Condition (i): No edge receives the same color as one of its endpoints.

If we only consider the total coloring of the graph G such that the Condition (v) is satisfied, then such a coloring is called an IE-total coloring of the graph G.

If f is an IE-total coloring of the graph G using k colors and for all $u, v \in V(G)$, $u \neq v$, we have $C(u) \neq C(v)$, then f is called a k-vertex-distinguishing IE-total coloring, or a k-VDIET coloring. The minimum number k for which G has a k-VDIET coloring is called the vertex-distinguishing IE-total chromatic number (or VDIET chromatic number) of the graph G and is denoted by $\chi^{ie}_{vt}(G)$.