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One Nonparabolic End Theorem on Kähler Manifolds

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Abstract: In this paper, the complete noncompact Kähler manifolds satisfying the weighted Poincaré inequality are considered and one nonparabolic end theorem which generalizes Munteanu's result is obtained.
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1 Introduction

Li and Wang[1–2] gave a rigidity theorem for a manifold M with $\lambda_1(M) > 0$ and the Ricci curvature Ric_M being bounded from below by $-\dfrac{n-1}{n-2}\lambda_1(M)$, where $\lambda_1(M)$ is the lowest bound of the spectrum of the Laplacian acting on L^2 functions. A complete Riemannian manifold $(M, \mathrm{d}s^2)$ is said to satisfy a weighted Poincaré inequality with non-negative weighted function ρ if the equality

$$\int_M \rho\phi^2 \leq \int_M |\nabla\phi|^2$$

holds for all compactly supported piecewise smooth functions $\phi \in C_0^{+\infty}(M)$, where $C_0^{+\infty}(M)$ are all compactly supported piecewise smooth functions on M. $(M, \mathrm{d}s^2)$ is said to satisfy property $(\mathcal{P}_\rho)$ if M satisfies a weighted Poincaré inequality with ρ and $\rho\mathrm{d}s^2$ being complete. Obviously, the notion of the property $(\mathcal{P}_\rho)$ is a generalization of $\lambda_1(M) > 0$. Li and Wang considered a class of Riemannian manifolds of dimension $n \geq 4$ satisfying the property $(\mathcal{P}_\rho)$ and having the Ricci curvature bounded below in terms of the weight function and gave a

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rigidity theorem if ρ grows no more than exponential increase (see Theorem 5.2 in [3]). Later, Cheng and Zhou[4] obtained a result which generalized Theorem 5.2 in [3]. Considering the manifolds satisfying property $(\mathcal{P}_\rho)$ for a weight function ρ that has limit zero at infinity, i.e., $\lim_{x\to+\infty}\rho(x)=0$, Li and Wang[3] proved that for $n\geq 4$ and the Ricci curvature $Ric_M(x)$ of M being bounded from below by $-\dfrac{n-1}{n-2}\rho(x)$ for each $x\in M$, the Riemannian manifold has only one nonparabolic end. In the setting of Kähler manifolds case, Munteanu[5] obtained analogous result to that for Riemannian manifolds, i.e., a Kähler manifold M^{2n} $(n\geq 2)$ has only one nonparabolic end if the Ricci curvature $Ric_M(x)$ of M is bounded from below by $-4\rho(x)$ for each $x\in M$.

In this paper, the case of Kähler manifold is considered. Following the idea of Cheng and Zhou[4], a generalized theorem which contains the result of Munteanu[5] is obtained. More precisely, we have

Theorem 1.1 *Let M be a complete noncompact real $2n$-dimensional Kähler manifold with property $(\mathcal{P}_\rho)$ $(n\geq 2)$. Suppose that the Ricci curvature of M satisfies*

$$Ric_M(x)\geq -4\tau(x),$$

where the non-negative bounded function $\tau(x)$ satisfies Poincaré's inequality

$$\int_M \tau\phi^2 \leq \int_M |\nabla\phi|^2$$

for all $\phi\in C_0^{+\infty}(M)$. If

$$\lim_{x\to\infty}\sup\{\rho(x),\ \tau(x)\}=0,$$

then M has only one nonparabolic end.

Remark 1.1 If we choose $\tau=\rho$ in Theorem 1.1, then it is just Munteanu's result, that is, Theorem 1 in [5].

2 Proof of the Main Result

Assume by absurd that M had at least two non parabolic ends, and thus there would exist a bounded harmonic function f with finite Dirichlet integral on M (see [6]). Moreover, we can assume that $\inf f=0$ and $\sup f=1$ with infimum and supremum achieved at infinity of nonparabolic end E and $F=M\setminus E$, respectively. One has the improved Bochner formula for the function f:

$$\Delta h\geq -2\tau h+h^{-1}|\nabla h|^2, \tag{2.1}$$

where $h=|\nabla f|^{\frac{1}{2}}$ (see [7]).

Lemma 2.1 *Let $g=h\varphi(f)$ with $\varphi:(0,1)\to(0,+\infty)$ being a C^∞ function which is to be determined later. Then*

$$\Delta g\geq -2\tau g+g^{-1}|\nabla g|^2+g|\nabla f|^2(\varphi^{-1}\varphi''-\varphi^{-2}(\varphi')^2). \tag{2.2}$$