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Necessary Maximum Principle of Stochastic Optimal Control with Delay and Jump Diffusion

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Abstract: In this paper, we have studied the necessary maximum principle of stochastic optimal control problem with delay and jump diffusion.

Key words: stochastic differential equation, jump diffusion, delay, necessary maximum principle

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1 Introduction

The optimal control theory of stochastic problem has been widely used in economy, finance *etc*. The delay variables and jump diffusions make the problem more complex than the one with only Brownian motion. Actually, delay and jump are very common in real world, and its optimal control problem is very challenging and meaningful. In this paper, we study the necessary maximum principle of stochastic optimal control problem with delay and jump diffusion.

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In last forty years, the maximum principle of stochastic problem has been studied by many researchers. The research started by the work of Kushner[1] and Bismut[2], and improved by Bensoussan[3], Haussmann[4], and Peng[5], Yong and Zhou[6] reviewed and improved the previous work based on these famous works. These works are mainly about the Wiener process.

In last decade, the studies about stochastic optimal problem with delay have developed very fast. In 2000, Øksendal and Sulem[7] obtained two sufficient maximum principles of stochastic optimal problem with delay, and cooperated with Elsanousi *et al.*[8] to give the solvability result of optimal control for random harvest problem with delay. Later, Larssen[9] applied the dynamic programming method to solve stochastic optimal control problem with delay. In 2010, based on Peng and Yang's work (see [10]), Chen and Wu[11] gave the necessary and sufficient principles of the stochastic problem with delay. There were also many works for the optimal control problems with the jump diffusion. Tang and Li[12] obtained the necessary maximum principle about this problem, Framstad *et al.*[13] gave the sufficient maximum principle. Also, in 2011, Øksendal *et al.*[14] gave a result about maximum principle on stochastic system with delay and jump, and our result is different with this result in model and proof of the theorems, such as higher dimensions, different parameters, and the proof details.

In Section 2, we introduce the preliminaries about the stochastic differential equation with delay and jump diffusion, and give several assumptions which should be satisfied in the following results. In Section 3, with the aid of Itô's formula and variational technique, we obtain the necessary maximum principle for the stochastic problem with delay and jump diffusion.

2 Preliminaries

Firstly, we introduce the stochastic differential equation with delay and jump diffusion. Let the state variable $X(t)$ be a jump diffusion which is affected by the control variable $v(t)$ and its delay variable $v(t-\delta)$. Then the equation of $X(t)$ can be described as follows:

$$\begin{aligned}\mathrm{d}X(t) &= b(t, X(t), X(t-\delta), v(t), v(t-\delta))\mathrm{d}t + \sigma(t, X(t), X(t-\delta), v(t), v(t-\delta))\mathrm{d}B(t)\\ &\quad + \int_{\mathbf{R}^n} \theta(t, X(t), X(t-\delta), v(t), v(t-\delta), z)\tilde{N}(\mathrm{d}t, \mathrm{d}z), \qquad t\in[0, T], \qquad (2.1)\end{aligned}$$

$$X(t)=\alpha(t), \quad v(t)=\beta(t), \qquad t\in[-\delta, 0]. \qquad (2.2)$$

Here, $b:[0, T]\times\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}^k\times\mathbf{R}^k\longrightarrow\mathbf{R}^n$, $\sigma:[0, T]\times\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}^k\times\mathbf{R}^k\longrightarrow\mathbf{R}^{n\times d}$ and $\theta:[0, T]\times\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}^k\times\mathbf{R}^k\times\mathbf{R}\longrightarrow\mathbf{R}^{n\times l}$ are known functions, $(B(t))_{t\in[0, T]}$ is a d dimensional Brownian motion on $(\Omega, \mathscr{F}, P, \{\mathscr{F}_t\}_{t\geq 0})$,

$$\begin{aligned}\tilde{\boldsymbol{N}}(\mathrm{d}t, \mathrm{d}z) &= (\tilde{N}_1(\mathrm{d}t, \mathrm{d}z), \cdots, \tilde{N}_l(\mathrm{d}t, \mathrm{d}z))^{\mathrm{T}}\\ &= (N_1(\mathrm{d}t, \mathrm{d}z)-\lambda_1(\mathrm{d}z)\mathrm{d}t, \cdots, N_l(\mathrm{d}t, \mathrm{d}z)-\lambda_l(\mathrm{d}z)\mathrm{d}t)^{\mathrm{T}}.\end{aligned}$$

We define

$$\boldsymbol{\eta}(t)=(\eta_1(t), \cdots, \eta_l(t)),$$