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Strong Convergence Theorems for Mixed Type Asymptotically Nonexpansive Mappings

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Abstract: The purpose of this paper is to study a new two-step iterative scheme with mean errors of mixed type for two asymptotically nonexpansive self-mappings and two asymptotically nonexpansive nonself-mappings and prove strong convergence theorems for the new two-step iterative scheme in uniformly convex Banach spaces.

Key words: mixed type asymptotically nonexpansive mapping, uniformly convex Banach space, common fixed point, strong convergence
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1 Introduction

Let K be a nonempty subset of a real normed linear space E. A mapping $T : K \to K$ is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim\limits_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \qquad x, y \in K,\ n \geq 1. \tag{1.1}$$

The class of asymptotically nonexpansive self-mappings was introduced by Goebel and Kirk[1] in 1972, as an important generalization of the class of nonexpansive self-mappings, who proved that if K is a nonempty closed convex subset of a real uniformly convex Banach space E and T is an asymptotically nonexpansive self-mapping of K, then T has a fixed point.

Since then, some authors proved weak and strong convergence theorems for asymptotically nonexpansive self-mappings in Banach spaces (see [2–6]), which extended and improved the result of Goebel and Kirk in several ways.

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In 2003, Chidume *et al.*[7] introduced the concept of asymptotically nonexpansive nonself-mappings, which is a generalization of an asymptotically nonexpansive self-mapping.

Definition 1.1[7] *Let K be a nonempty subset of a real normed linear space E. Let $P : E \to K$ be a nonexpansive retraction of E onto K. A nonself-mapping $T : K \to E$ is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim\limits_{n\to\infty} k_n = 1$ such that*

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq k_n\|x - y\|, \qquad x, y \in K,\ n \geq 1. \tag{1.2}$$

Let K be a nonempty closed convex subset of a real uniformly convex Banach space E. Chidume *et al.*[7] studied the following iteration scheme:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1 - a_n)x_n + a_nT(PT)^{n-1}x_n), \qquad n \geq 1, \end{cases} \tag{1.3}$$

and proved some strong and weak convergence theorems for an asymptotically nonexpansive nonself-mapping.

In 2006, Wang[8] generalized the iteration process (1.3) as follows:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1 - a_n)x_n + a_nT_1(PT_1)^{n-1}y_n), \\ y_n = P((1 - b_n)x_n + b_nT_2(PT_2)^{n-1}x_n), \qquad n \geq 1, \end{cases} \tag{1.4}$$

and proved some strong and weak convergence theorems for two asymptotically nonexpansive nonself-mappings.

In 2012, Guo *et al.*[9] generalized the iteration process (1.4) as follows:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = P((1 - a_n)S_1^nx_n + a_nT_1(PT_1)^{n-1}y_n), \\ y_n = P((1 - b_n)S_2^nx_n + b_nT_2(PT_2)^{n-1}x_n), \qquad n \geq 1, \end{cases} \tag{1.5}$$

where S_1, $S_2 : K \to K$ are two asymptotically nonexpansive self-mappings and T_1, $T_2 : K \to E$ are two asymptotically nonexpansive nonself-mappings and $\{a_n\}$, $\{b_n\}$ are real sequences in $[0, 1)$, and proved some strong convergence theorems for mixed type asymptotically nonexpansive mappings.

The purpose of this paper is to construct a new iteration scheme with mean errors of mixed type for two asymptotically nonexpansive self-mappings and two asymptotically nonexpansive nonself-mappings and prove some strong convergence theorems for the new two-step iterative scheme in uniformly convex Banach spaces. The results presented in the paper improve and extend some results in [9].

2 Preliminaries

Let E be a real Banach space, K be a nonempty closed convex subset of E and $P : E \to K$ be a nonexpansive retraction of E onto K. Let S_1, $S_2 : K \to K$ be two asymptotically nonexpansive self-mappings, and T_1, $T_2 : K \to E$ be two asymptotically nonexpansive nonself-mappings. Then we define the new iteration scheme of mixed type with mean errors