Communications
in
Mathematical
Research
31(3)(2015), 222–228

Globals of Completely Regular Monoids

Wu Qian-qian and Gan Ai-ping*
(*College of Mathematics and Information Science, Jiangxi Normal University, Nanchang, 330022*)

Communicated by Du Xian-kun

Abstract: An element of a semigroup S is called irreducible if it cannot be expressed as a product of two elements in S both distinct from itself. In this paper we show that the class $\mathcal{C}$ of all completely regular monoids with irreducible identity elements satisfies the strong isomorphism property and so it is globally determined.

Key words: completely regular monoid, irreducible element, power semigorup
2010 MR subject classification: 06A12
Document code: A
Article ID: 1674-5647(2015)03-0222-07
DOI: 10.13447/j.1674-5647.2015.03.04

1 Introduction and Preliminaries

The power semigroup, or global, of a semigroup S is the semigroup $P(S)$ of all nonempty subsets of S equipped with the multiplication

$$AB = \{ab : a \in A,\ b \in B\}, \qquad A, B \in P(S).$$

A class $\mathcal{K}$ of semigroups is said to be globally determined if any two members of $\mathcal{K}$ having isomorphic globals must themselves be isomorphic.

Tamura[1] asked in 1967 whether the class of all semigroups is globally determined. The question was negatively answered in the class of all semigroups by Mogiljanskaja[2] in 1973. Crvenković *et al.*[3] proved that involution semigroups are not globally determined in 2001. But it is known that the following classes are globally determined: groups [4–5]; rectangular groups [6]; completely 0-simple semigroups [7]; finite semigroups [8]; lattices and semilattices [9–10], finite simple semigroups and semilattices of torsion groups in which semilattices are finite [11]; completely regular periodic monoids with irreducible identity [12].

In this paper we devote to the study of the global determinacy of completely regular semigroup and show that the class of all completely regular monoids with irreducible identity

Received date: Aug. 3, 2013.
Foundation item: The NSF (11261021) of China and the NSF (20142BAB201002) of Jiangxi Province.
* **Corresponding author**.
E-mail address: molihuaqian@163.com (Wu Q Q), ganaiping78@163.com (Gan A P).

element satisfies the strong isomorphism property and so it is globally determined. Recall that a class $\mathcal{K}$ of semigroups is said to satisfy the strong isomorphism property if for any S, $S' \in \mathcal{K}$ and for every isomorphism ψ from $P(S)$ to $P(S')$, $\psi|_S$ (the restriction of ψ to S) is an isomorphism from S to S', where S (resp. S') is considered to be a subset of $P(S)$ (resp. $P(S')$) by identifying an element x of S (resp. S') with the singleton set $\{x\}$. It was proved by Kobayashi[10] that the class of semilattices satisfies the strong isomorphism property.

A few words on notation and terminology are in order. The set of idempotents of a semigroup S is denoted by $E(S)$, and for each $a \in S$ the $\mathcal{H}$-class of S containing a is denoted by $H_a(S)$. A singleton member of $P(S)$ frequently is identified with the element it contains. An element $a \in S$ is termed irreducible if it cannot be expressed as a product of two elements in S both distinct from themselves, i.e., if there exist $b, c \in S$ such that $a = bc$, then $a = b$ or $a = c$. It is clear that if S is a monoid whose identity element 1 is irreducible, then $1 = bc$ implies that $1 = b = c$.

We refer to the books [13–16] for all background information concerning semigroups and universal algebra.

2 Main Results

In this section, we denote the class of all completely regular monoids with irreducible identity elements by $\mathcal{C}$. For convenience, for every monoid S, we use 1 rather than 1_S to denote its identity element.

Our main result, establishing the global determinacy of $\mathcal{C}$, is proved via a sequence of lemmas. The first lemma implies that the class of all groups is globally determined, which is taken from Gould and Iskra[11–12].

Lemma 2.1[11] *Let S be a semigroup and $e \in E(S)$. Then $H_e(\mathcal{P}(S)) = H_e(S)$.*

Lemma 2.2 *Let $S \in \mathcal{C}$, and let A be an idempotent in $P(S)$. Then $|A| > 1$ if and only if A satisfies one of the following two statements*:

(i) *$A = \{1, a\}B$ or $A = B\{1, a\}$ for some $B \in P(S) \setminus \{A\}$ and some $a \in A$*;

(ii) *$A = C^2$ for some $C \in P(S)$ such that $C \neq C^3$.*

Proof. It follows immediately from Lemma 2.2 in [12].

Lemma 2.3[12] *Let $S \in \mathcal{C}$. For $A \in P(S)$ the following statements are equivalent*:

(i) *There exists an $e \in E(S)$ such that $A = \{1, e\}$*;

(ii) *A is idempotent and irreducible, and the only solutions to $XA = A$ are $\{1\}$ and A.*

Lemma 2.4 *Let $S \in \mathcal{C}$, and $A \in P(S)$. Then there exist $a \in S \setminus E(S)$ and $a^2 \in E(S)$ such that $A = \{1, a\}$ if and only if A satisfies the following statements*:

(i) *A^2 is idempotent, and A is not idempotent*;

(ii) *There is a unique $e \in E(S) \setminus \{1\}$ such that $A^2\{1, e\} = A^2$*;

(iii) *$A\{1, e\} = A^2$, where e is given as in* (ii);