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A Mean-variance Problem in the Constant Elasticity of Variance (CEV) Model

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Abstract: In this paper, we focus on a constant elasticity of variance (CEV) model and want to find its optimal strategies for a mean-variance problem under two constrained controls: reinsurance/new business and investment (no-shorting). First, a Lagrange multiplier is introduced to simplify the mean-variance problem and the corresponding Hamilton-Jacobi-Bellman (HJB) equation is established. Via a power transformation technique and variable change method, the optimal strategies with the Lagrange multiplier are obtained. Final, based on the Lagrange duality theorem, the optimal strategies and optimal value for the original problem (i.e., the efficient strategies and efficient frontier) are derived explicitly.

Key words: constant elasticity of variance model, mean-variance, optimal strategy

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1 Introduction

In this paper, we study a constant elasticity of variance (CEV) model. The CEV model is an extension model of the geometric Brownian motion (GBM). It was developed by [1–2], and then it was usually applied to calculating the option pricing problems (see [3–7]). In recent years, the CEV model began to apply in optimal investment research (see [8–10]).

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The study of mean-variance portfolio selection model should date back to Markowitz's Nobel Prize-winning work (see [11]). In this paper, we introduce the dynamic mean-variance problem to the CEV model. And the aim is to get the optimal strategy of this problem under two constrained controls: reinsurance/new business and investment (no-shorting) as stated by Bai and Zhang[12]. They studied the mean-variance problem under two types of risk models: a classical model and its diffusion approximation. They constructed a continuously differentiable function to get a viscosity solution and gave a verification theorem. Because of the appearance of the constant elasticity of variance, the approach to construct a viscosity solution cannot be used in our model. Fortunately, via the power transformation technique and variable change method proposed by Cox[4], we obtain the optimal strategy with the Lagrange multiplier for our model. And we can find the Theorem 2.2 in [12] is just our case when the constant elasticity of variance is zero. Finally, based on the Lagrange duality theorem in [13], the optimal strategy and optimal value for the original problem (i.e., the efficient strategy and efficient frontier) are derived explicitly.

The rest of this paper is organized as follows. Section 2 presents the CEV model which we focus on. In Section 3, the optimization problem is developed. We get the optimal strategy in Section 4 and find the efficient strategy and efficient frontier in Section 5.

2 The Model

Let the claim process be denoted by

$$\mathrm{d}C(t) = a\mathrm{d}t - b\mathrm{d}W^0(t), \tag{2.1}$$

where a and b are positive constants, $W^0(t)$ is a standard Brownian motion. Assume that the premium is $c_0 = (1+\eta)a$ with safety loading $\eta > 0$.

Let $u(t) \geq 0$ denote the retention level of reinsurance/new business ($u(t) \in [0,\, 1]$ corresponded to a proportional reinsurance and $u(t) > 1$ corresponded to acquiring new business). The insurer pays reinsurance premium at the rate $c_1 = (1+\theta)a(1-u(t))$ with safety loading $\theta > \eta > 0$. So the surplus process is given by the dynamics

$$\begin{aligned}\mathrm{d}R(t) &= c_0\mathrm{d}t - u(t)\mathrm{d}C(t) - c_1\mathrm{d}t \\ &= (\eta - \theta(1-u(t))a)\mathrm{d}t + bu(t)\mathrm{d}W^0(t).\end{aligned} \tag{2.2}$$

In addition, the insurer is supposed to invest its surplus in a risk-free asset (bond or bank account) and a risky asset. Let $S_0(t)$ denote the price process of the risk-free asset given by

$$\mathrm{d}S_0(t) = r_0 S_0(t)\mathrm{d}t, \qquad r_0 > 0. \tag{2.3}$$

Let $S_1(t)$ denote the price process of the risky asset described by a CEV model

$$\mathrm{d}S_1(t) = r_1 S_1(t)\mathrm{d}t + \sigma S_1^{\beta+1}(t)\mathrm{d}W^1(t), \tag{2.4}$$

where $r_1 > r_0$ is the expected instantaneous rate of the risky asset, $\sigma S_1^{\beta}(t)$ is the instantaneous volatility and $\beta \leq 0$ is the elasticity parameter. $\{W^0(t)) : t \geq 0\}$ and $\{W^1(t)) : t \geq 0\}$ are standard Brownian motions defined on a complete probability space $(\Omega,\, \mathcal{F},\, P)$. The filtration $F = \{\mathcal{F}_t\}$ is right continuous filtration of sigma-algebras on this space and denotes