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Multilinear Fractional Integral Operators on Morrey Spaces with Variable Exponent on Bounded Domain

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Abstract: We prove the boundedness of multilinear fractional integral operators on products of the variable exponent Morrey spaces on bounded domain.

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1 Introduction

Let Ω be an open set in the n-dimensional Euclidean space $\mathbf{R}^n$ with $|\Omega| > 0$, and
$$\Omega^m = \Omega \times \Omega \times \cdots \times \Omega$$
be the m-fold product space of Ω. The multilinear fractional integrals on Ω are defined by
$$I_{\beta,m}(f_1, \cdots, f_m)(x) = \int_{\Omega^m} \frac{f_1(y_1)\cdots f_m(y_m)}{|(x-y_1, \cdots, x-y_m)|^{mn-\beta}} \mathrm{d}y_1 \cdots \mathrm{d}y_m,$$
where $0 < \beta < mn$, $|(x-y_1, \cdots, x-y_m)| = \sqrt{|x-y_1|^2 + \cdots + |x-y_m|^2}$.

Let $\Omega = \mathbf{R}^n$, $I_{\beta,1} = I_\beta$ with $I_\beta f(x) = \displaystyle\int_{\mathbf{R}^n} \frac{f(y)}{|x-y|^{n-\beta}} \mathrm{d}y$. The famous Hardy-Littlewood-Sobolev theorem tells us that the fractional integral operator I_β is a bounded operator from the usual Lebesgue spaces $L^{p_1}(\mathbf{R}^n)$ to $L^{p_2}(\mathbf{R}^n)$ where $1 < p_1 < p_2 < \infty$ and $\dfrac{1}{p_1} - \dfrac{1}{p_2} = \dfrac{\beta}{n}$. Kenig and Stein[1] as well as Grafakos and Kalton[2] considered the boundedness

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of a family of related multilinear fractional integrals. Moen[3] established some weighted inequalities for multilinear fractional integral operators. Hu and Lin[4] got weighted norm inequalities for multilinear singular integral operators and applications. Tao and He[5] proved the boundedness of multilinear operators on generalized Morrey spaces over the quasi-metric space of non-homogeneous type. Also, many results about multilinear fractional integrals on Morrey spaces have been studied, (see [6–7]). Especially, Tang[7] presented the boundedness of multilinear fractional integral operators on Morrey spaces. One of his results is rewrited as the following theorem:

Theorem 1.1[7] *Suppose that* $m \in \mathbf{N}$, $0 < \beta < mn$,

$$\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_m} - \frac{\beta}{n}$$

and

$$\frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2} + \cdots + \frac{1}{q_m} - \frac{\beta}{n}$$

with $1 < p_i \leq q_i < \infty$ *for* $i = 1, 2, \cdots, m$. *Then there exists a constant* $C > 0$ *such that*

$$\|I_{\beta,m}(f_1, f_2, \cdots f_m)\|_{\mathcal{M}_p^q(\mathbf{R}^n)} \leq C \prod_{i=1}^{m} \|f_i\|_{\mathcal{M}_{p_i}^{q_i}(\mathbf{R}^n)}.$$

In this article, our main aim is to extend one of Tang's work to the variable exponent case. In recent twenty years, variable exponent spaces have been generating interest because of its connection with the study of variational integrals and partial differential equations with a non-standard growth condition (see [8–9]).

We first recall the definitions of Lebesgue spaces with variable exponent $L^{p(\,\cdot\,)}(\Omega)$ (see [10]).

Let $p(\,\cdot\,) : \Omega \to [1, \infty)$ be a measurable function. The variable exponent Lebesgue space $L^{p(\,\cdot\,)}(\Omega)$ is defined by

$$L^{p(\,\cdot\,)}(\Omega) := \left\{ f \text{ is measurable} : \int_\Omega \left| \frac{f(x)}{\lambda} \right|^{p(x)} \mathrm{d}x < \infty \text{ for some constant } \lambda > 0 \right\}.$$

The space $L^{p(\,\cdot\,)}_{\mathrm{loc}}(\Omega)$ is defined by

$$L^{p(\,\cdot\,)}_{\mathrm{loc}}(\Omega) := \{ f \text{ is measurable} : f \in L^{p(\,\cdot\,)}(K) \text{ for all compact subsets } K \subset \Omega \}.$$

$L^{p(\,\cdot\,)}(\Omega)$ is a Banach space with the norm defined by

$$\|f\|_{L^{p(\,\cdot\,)}(\Omega)} := \inf \left\{ \lambda > 0 : \int_\Omega \left| \frac{f(x)}{\lambda} \right|^{p(x)} \mathrm{d}x \leq 1 \right\}.$$

We denote

$$p_- := \operatorname*{ess\,inf}_{x\in\Omega} p(x), \qquad p_+ := \operatorname*{ess\,sup}_{x\in\Omega} p(x).$$

Let $\mathcal{P}(\Omega)$ be the set of measurable function $p(\,\cdot\,)$ on Ω with value in $[1, \infty)$ such that

$$1 < p_-(\Omega) \leq p(\,\cdot\,) \leq p_+(\Omega) < \infty.$$

We say that a function $p(\,\cdot\,) : \Omega \longrightarrow \mathbf{R}$ is locally log-Hölder continuous, and denote this by $p(\,\cdot\,) \in LH_0$, if there exists a constant C such that for all $x, y \in \Omega$, $|x - y| \leq \dfrac{1}{2}$,

$$|p(x) - p(y)| \leq \frac{-C}{\log(|x - y|)}.$$