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Almost Fuzzy Compactness in L-fuzzy Topological Spaces

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Abstract: In this paper, the notion of almost fuzzy compactness is defined in L-fuzzy topological spaces by means of inequality, where L is a completely distributive DeMorgan algebra. Its properties are discussed and many characterizations of it are presented.

Key words: L-fuzzy topological space, L-fuzzy almost compactness, L-fuzzy compactness, almost fuzzy compactness

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1 Introduction

Almost compactness is a very important concept. Many researchers have tried successfully to generalize the compactness theory of general topology to L-topology (see [1–9]). Recently, Shi[10] introduced new definitions of almost fuzzy compactness in L-topological spaces with the help of inequality, where L is a completely distributive DeMorgan algebra. The aim of this paper is to generalize the notion of almost compactness in [10] to L-fuzzy topological spaces, thus some properties and characterizations are researched.

2 Preliminaries

In this paper, $(L, \vee, \wedge, ')$ is a completely distributive DeMorgan algebra (i.e., completely distributive lattice with order-reversing involution, see [11]). The largest element and the

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smallest element in L are denoted by $\top$ and $\bot$, respectively.

Definition 2.1[12] *An L-fuzzy topology on a set X is a map* $\tau : L^X \to L$ *such that*

(1) $\tau(\underline{\top}) = \tau(\underline{\bot}) = \top$;

(2) *for all* $U, V \in L^X$, $\tau(U \wedge V) \geqslant \tau(U) \wedge \tau(V)$;

(3) *for all* $U_j \in L^X$, $j \in J$, $\tau(\bigvee\limits_{j\in J} U_j) \geqslant \bigwedge\limits_{j\in J} \tau(U_j)$.

The pair (X, τ) is called an L-fuzzy topological space. Generally, $\tau(U)$ can be regarded as degree to which $U \in L^X$ is an open set, is called the degree of openness of U, meanwhile, $\tau^*(U) = \tau(U')$ is called the degree of closedness of U. For all $\mathcal{U} \subseteq \mathcal{L}^{\mathcal{X}}$, $\tau(\mathcal{U}) = \bigwedge\limits_{\mathcal{A}\in\mathcal{U}} \tau(\mathcal{A})$ is called the degree of openness of $\mathcal{U}$.

For a subfamily $\Phi \subseteq L^X$, $2^{(\Phi)}$ denotes the set of all finite subfamilies of Φ.

Definition 2.2[13] *A map* $\widetilde{\subset} : L^X \times L^X \to L$ *is an L-fuzzy inclusion on X, it is defined as* $\widetilde{\subset}(A, B) = \bigwedge\limits_{x\in X} (A'(x) \vee B(x))$. *For simplicity, it is denoted by* $[A\widetilde{\subset}B]$ *instead of* $\widetilde{\subset}(A, B)$, *i.e.,* $[A\widetilde{\subset}B] = \bigwedge\limits_{x\in X} (A'(x) \vee B(x))$.

3 Definitions and Properties of L-fuzzy Almost Compactness

Definition 3.1 *Let* (X, τ) *be an L-fuzzy topological space and* $A \in L^X$. *For all* $r \in L$,

$$A^{\circ}_{\tau_r} = \bigvee\{B \mid B \leq A,\ r \leq \tau(B),\ B \in L^X\}$$

is called the r-interiors of A with respect to τ. *The r-closures of A with respect to* τ *is defined as*

$$A^{-}_{\tau_r} = \bigwedge\{B \mid A \leq B,\ r \leq \tau^*(B),\ B \in L^X\}.$$

In the following part, $\{A \mid A^{\circ}_{\tau_r} = A,\ A \in L^X\}$ is marked as τ_r, i.e.,

$$\tau_r = \{A \mid A^{\circ}_{\tau_r} = A,\ A \in L^X\}.$$

An L-topology $\mathcal{T}$ can be regarded as a map $\chi_{\mathcal{T}} : L^X \to L$ defined by

$$\chi_{\mathcal{T}}(A) = \begin{cases} \top, & A \in \mathcal{T}; \\ \bot, & A \notin \mathcal{T}. \end{cases}$$

In this way, $(X, \chi_{\mathcal{T}})$ is a special L-fuzzy topological space and

$$A^{-}_{\chi_{\mathcal{T}_\top}} = A^{-}, \quad A^{\circ}_{\chi_{\mathcal{T}_\top}} = A^{\circ}, \qquad A \in L^X.$$

This shows that Definition 3.1 can be regarded as the generalization in L-fuzzy topological space of the interiors and closures in L-topological space.

By Definition 3.1, we have the following theorem.

Theorem 3.1 *Let* $(X, \mathcal{T})$ *be an L-topological space and* $A \in L^X$. *Then, for all* $r, s \in L$,

(1) $A \leq A^{-}_{\tau_r}$, $\tau^*(A) \leq \tau^*(A^{-}_{\tau_r})$;

(2) $A^{\circ}_{\tau_r} \leq A$, $\tau(A) \leq \tau(A^{\circ}_{\tau_r})$;