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Global Dynamics of a Predator-prey Model

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Abstract: In this paper, we consider a predator-prey model. A sufficient condition is presented for the stability of the equilibrium, which is different from the one for the model with Hassell-Varley type functional response. Furthermore, by constructing a Lyapunov function, we prove that the positive equilibrium is asymptotically stable.

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1 Introduction

The Allee effect describes a scenario in which populations at low numbers are affected by a positive relationship between the per capita population growth rate and density, which increases their likelihood of extinction. Odum and Allee[1] got an approach of the model with Allee effect of single

$$x'(t) = xg(x), \tag{1.1}$$

where $g(x)$ denotes the density-dependent per capita growth rate. In (1.1), three basic scenarios may occur (see [2]):

1. Unconditional extinction. If g is negative for all x, we say that the Allee effect is too strong. In this situation, populations go inevitably extinct regardless of their initial sizes;

2. Extinction-survival. If g is positive for intermediate value but negative for very low or high values of x. In this situation, two equilibria emerge. It is so-called the model with strong Allee effect;

3. Unconditional survival. By weakening the influence of the Allee effect, g is always positive. In this situation, the population can reach the carrying capacity.

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With the strong Allee effect, the following growth model for one species was proposed in [3]

$$x'(t) = ax\Big(1 - \frac{x}{K}\Big)\Big(\frac{x}{S} - 1\Big), \tag{1.2}$$

where a is the organic growth of the prey, $S > 0$ a critical population threshold, $K > S$ the carrying capacity of the prey in the absence of the predator. A general two-component predator-prey model is of the following form (see [4])

$$\begin{cases} x'(t) = xg(x) - yp(x), \\ y'(t) = y\Big(h - n\dfrac{y}{x}\Big), \end{cases} \tag{1.3}$$

where $x(t)$ and $y(t)$ stand for the population densities of the prey and the predator at time $t > 0$ respectively, $g(x)$ the per capita growth rate of the prey in the absence of the predator, $p(x)$ the so-called predator functional response to the prey, $h > 0$ the intrinsic growth rates of the predator, $n > 0$ a measure of the food quality that the prey provides for conversion into predator birth.

Another predator-prey model with Hassell-Varley type functional response takes the following form (see [5])

$$\begin{cases} x'(t) = ax\Big(1 - \dfrac{x}{K}\Big) - \dfrac{bxy}{\beta y^\gamma + x}, \\ y'(t) = y\Big(\dfrac{bex}{\beta y^\gamma + x} - D\Big), \\ x(0) > 0, \quad y(0) > 0, \end{cases} \tag{1.4}$$

where γ is called the Hassell-Varley constant with $\gamma \in (0, 1)$, $D > 0$ the death rate of the predator, $b > 0$ the intrinsic growth rate of the predator, $\beta > 0$ the half saturation constant, $e > 0$ the conversion factor denoting the number of newly born predators for each captured prey. Later, Yu and Sun*et al.*[6] considered the following more general predator-prey model incorporating a constant prey refuge with Hassell-Varley type functional response

$$\begin{cases} x'(t) = ax\Big(1 - \dfrac{x}{K}\Big) - \dfrac{b(x-m)y}{y^\gamma + c(x-m)}, \\ y'(t) = y\Big(\dfrac{be(x-m)}{y^\gamma + c(x-m)} - D\Big), \\ x(0) > m, \quad y(0) > 0. \end{cases} \tag{1.5}$$

They proved that (1.5) admits a globally asymptotically stable equilibrium provided that

$$1 + \gamma\delta d - \gamma\delta - \frac{2m}{K} \le 0, \tag{1.6}$$

where $\delta = \dfrac{be}{ac}$, $d = \dfrac{Dc}{be}$.

Inspired by (1.2)–(1.5), in this paper, we consider the following predator-prey model with strong Allee effect:

$$\begin{cases} x'(t) = ax\Big(1 - \dfrac{x}{K}\Big)\Big(\dfrac{x}{S} - 1\Big) - \dfrac{b(x-m)y}{y^\gamma + c(x-m)}, \\ y'(t) = y\Big(\dfrac{be(x-m)}{y^\gamma + c(x-m)} - D\Big), \end{cases} \tag{1.7}$$

where $m > 0$ is a constant denoting the number of the prey using refuges, which protects m of the prey from the predation, $c > 0$ is a constant. In this paper, we show that under