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Exact Controllability for a Class of Nonlinear Evolution Control Systems

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Abstract: In this paper, we study the exact controllability of the nonlinear control systems. The controllability results by using the monotone operator theory are established. No compactness assumptions are imposed in the main results.

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1 Introduction

In this paper, we study the exact controllability of the following nonlinear systems:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t, x(t), u(t)) + B(t)u(t), \\ x(0) = 0, \qquad t \in J, \end{cases} \tag{1.1}$$

where $J = [0, T]$, the state of the system $x \in X$ and $u(t) \in V$ is the control at time t, X and V are Hilbert spaces. Let $Y = L^2(0, T; X)$ be the solution space and $U = L^2(0, T; V)$ be the control function space. The nonlinear function $f : J \times X \times V \to X$, $A(t)$ is a linear operator on X for each t, $B(t) : V \to X$ is a bounded linear operator.

We know that the linear system

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t), \\ x(t_0) = x_0, \qquad t \in J \end{cases}$$

can be applied to many fields such as engineering, economic growth model, continuous mechanics and so on. Many properties and classical results have been obtained based on researching the linear system. However, the nonlinear systems (1.1) are more common in practical problems. Naturally, people want to know: if a controllable linear system is

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perturbed by a nonlinear perturbation $f(t, x, u)$ and becomes a nonlinear system

$$\dot{x}(t) = A(t)x(t) + f(t, x(t), u(t)) + B(t)u(t),$$

then will the perturbed system preserve the controllability of the linear system? In 1972, Mirza and Womack[1] considered the nonlinear system above and obtained controllability. In the same year, Lukes[2] proved the controllability of this nonlinear system with A, B constant matrices. In 1976, Dauer[3] concluded that the system was completely controllable. Recently, many authors have studied the controllability to this system in infinite dimensional space (see [4–11]). In this paper, we present some abstract results for nonlinear control system with some useful corollaries.

The outlay of the paper is as follow. In Section 2, we give some preliminaries. We study the system (1.1) and obtain the controllability for this system in Section 3.

2 Preliminaries

Firstly, we introduce some notations, definitions and a lemma, which are used in this paper.

For (1.1), we suppose that $A(t)$ generates an evolution system $\{E(t, s)\}$, $f : J \times X \times V \to X$ is a nonlinear operator, measurable in the first argument and continuous in the last two arguments. $C(J, X)$ denotes the set of all continuous functions x from J to X with the usual maximum norm denoted by $\|\cdot\|_C$.

Definition 2.1 *A function $x \in C(J, X)$ is called a mild solution of* (1.1) *on J if*

$$x(t) = \int_0^t E(t, s)f(s, x(s), u(s))\mathrm{d}s + \int_0^t E(t, s)B(s)u(s)\mathrm{d}s, \qquad t \in J.$$

Definition 2.2 *The system* (1.1) *is said to be controllable on the interval $J = [0, T]$ if for every $x_1 \in X$, there exists a control $u \in U$ such that the mild solution $x(t)$ of* (1.1) *satisfies $x(0) = 0$ and $x(T) = x_1$.*

X is a Hilbert space with scalar product $\langle\, \cdot\, ,\, \cdot\, \rangle$. An operator $F : X \to X$ is called strongly monotone (or monotone) if there exists $\beta > 0$ (or $\beta = 0$) such that

$$\langle F(x) - F(y), x - y\rangle \geq \beta\|x - y\|^2, \qquad x, y \in X.$$

F is called hemicontinuous if $x \in X$, $h \in X$, $t > 0$, and if $t \to 0$ implies $F(x+th) \rightharpoonup F(x)$, here "$\rightharpoonup$" means weak convergence in X.

Lemma 2.1[12] *Suppose that $T : X \to X$ is maximal monotone and $\theta \in D(T)$, $P : X \to X$ is hemicontinuous, monotone, bounded and satisfies coercive condition, i.e.,*

$$\lim_{\|x\|\to+\infty} \frac{\langle Px, x\rangle}{\|x\|} = +\infty.$$

Then $T + P$ is onto.