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Dynamics of a Delayed Predator-prey System with Stage Structure for Predator and Prey

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Abstract: In this paper, a predator-prey system with two discrete delays and stage structure for both the predator and the prey is investigated. The dynamical behaviors such as local stability and local Hopf bifurcation are analyzed by regarding the possible combinations of the two delays as bifurcating parameter. Some explicit formulae determining the direction of the Hopf bifurcation and the stability of the bifurcating periodic solutions are derived by using the normal form method and the center manifold theory. Finally, numerical simulations are presented to support the theoretical analysis.

Key words: time delay, stage-structure, Hopf bifurcation, stability, periodic solution

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1 Introduction

Predator-prey systems have been studied by many authors due to their universal existence and importance (see [1–5]). It is well known that there are many species whose individual members have a life history that takes them through immature stage and mature stage. Based on this fact, stage-structured predator-prey systems have been investigated by many authors in recent years (see [6–13]). A stage-structured system of one species' growth consisting of immature and mature individuals was investigated by Aiello and Freedman[3]. Li F and Li H W[5] investigated a predator-prey system with stage structure for the prey:

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$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = ax_2(t) - r_1x_1(t) - bx_1(t), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = bx_1(t) - r_2x_2(t) - b_1x_2^2(t) - \dfrac{a_1x_2^2(t)y(t)}{1+mx_2^2(t)}, \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = \dfrac{a_2x_2^2(t-\tau)y(t-\tau)}{1+mx_2^2(t-\tau)} - ry(t), \end{cases} \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ represent the densities of the immature prey and the mature prey at time t, respectively. $y(t)$ represents the density of the predator at time t. They studied the Hopf bifurcation phenomenon and properties of periodic solutions of (1.1). Xu[6] considered the following predator-prey system with stage structure for the predator:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)(r - ax(t) - \dfrac{a_1y_2(t)}{1+mx(t)}), \\ \dfrac{\mathrm{d}y_1(t)}{\mathrm{d}t} = \dfrac{a_2x(t-\tau)y_2(t-\tau)}{1+mx(t-\tau)} - r_1y_1(t) - Dy_1(t), \\ \dfrac{\mathrm{d}y_2(t)}{\mathrm{d}t} = Dy_1(t) - r_2y_2(t), \end{cases} \tag{1.2}$$

where $x(t)$ represents the density of the prey at time t. $y_1(t)$ and $y_2(t)$ represent the densities of the immature predator and the mature predator at time t, respectively. He investigated the permanence and local and global stability of (1.2).

Motivated by the work above, and considering that both predator and prey have a life history that takes them through immature stage and mature stage, we propose the following delayed system with Holling-III functional response in the present paper:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = ax_2(t) - r_1x_1(t) - bx_1(t), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = bx_1(t) - r_2x_2(t) - b_1x_2(t)x_2(t-\tau_1) - \dfrac{a_1x_2^2(t)y_2(t)}{1+mx_2^2(t)}, \\ \dfrac{\mathrm{d}y_1(t)}{\mathrm{d}t} = \dfrac{a_2x_2^2(t-\tau_2)y_2(t-\tau_2)}{1+mx_2^2(t-\tau_2)} - d_1y_1(t) - cy_1(t), \\ \dfrac{\mathrm{d}y_2(t)}{\mathrm{d}t} = cy_1(t) - d_2y_2(t) - b_2y_2^2(t), \end{cases} \tag{1.3}$$

where $x_1(t)$ and $x_2(t)$ represent the densities of the immature prey and the mature prey at time t, respectively; $y_1(t)$ and $y_2(t)$ represent the densities of the immature predator and the mature predator at time t, respectively; a is the birth rate of the immature prey; b and c are the rates of immature individuals becoming mature individuals for the prey and the predator, respectively; a_1 is the capturing rate of the mature predator; $\dfrac{a_2}{a_1}$ is the rate conversing the mature prey into the new immature predator; b_1 and b_2 are the intraspecific competition rates of the mature prey and the mature predator, respectively; r_1 and r_2 are the death rates of the immature prey and the mature prey, respectively; d_1 and d_2 have the similar meanings to r_1 and r_2; $\dfrac{a_1x_2^2(t)}{1+mx_2^2(t)}$ is Holling-III functional response which describes the consumption of the mature prey by the mature predator. All the parameters in (1.3)