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A New Generalized FB Complementarity Function for Symmetric Cone Complementarity Problems

Zhang Yun-sheng and Gao Lei-fu*
(*College of Science, Liaoning Technical University, Fuxin, Liaoning, 123000*)

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Abstract: We establish that the generalized Fischer-Burmeister(FB) function and penalized Generalized Fischer-Burmeister (FB) function defined on symmetric cones are complementarity functions (C-functions), in terms of Euclidean Jordan algebras, and the Generalized Fischer-Burmeister complementarity function for the symmetric cone complementarity problem (SCCP). It provides an affirmative answer to the open question by Kum and Lim (Kum S H, Lim Y. Penalized complementarity functions on symmetric cones. *J. Glob. Optim.*. 2010, **46**: 475–485) for any positive integer.

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1 Introduction

The symmetric cone complementarity problem (SCCP) is defined to find $x, y \in V$ such that

$$\boldsymbol{x} \in K, \quad \boldsymbol{y} = f(x) \in K, \quad \langle \boldsymbol{x}, \ \boldsymbol{y} \rangle = 0, \tag{1.1}$$

where K is the cone of squares in Euclidean Jordan algebra V, and $F : V \to V$ is a continuously differentiable mapping (see [1]–[2]). This class of problem provides a unified framework for the classical nonlinear and complementarity problem (NCP), the second-order cone optimization and complementarity problem (SCOCP), and the semi-definite programming and complementarity problem (SDCP), and has attracted much attention due to its various applications in operations research, economics and engineering.

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* **Corresponding author**.
E-mail address: zhangysbad@126.com (Zhang Y S), gaoleifu@163.com (Gao L F).

A popular and powerful approach to solve the complementarity problem is to reformulate each problem as an equivalent system of non-smooth equations by a complementarity function (C-function) (see [3]–[4]) or as an unconstrained minimization problem by merit function (M-function) (see [5]). A function $\phi : V \times V \to V$ is called a C-function for SCCP if

$$\phi(\boldsymbol{x},\ \boldsymbol{y}) = 0 \Leftrightarrow \boldsymbol{x} \in K,\ \boldsymbol{y} = f(\boldsymbol{x}) \in K,\ \langle \boldsymbol{x},\ \boldsymbol{y}\rangle = 0. \tag{1.2}$$

Various C-functions for the standard NCP functions ware extend to the SCCP. For instance, Gowda *et al.*[6] showed that the Fischer-Burmeister function

$$\boldsymbol{\phi}_{\rm FB}(\boldsymbol{x},\ \boldsymbol{y}) = \boldsymbol{x} + \boldsymbol{y} - (\boldsymbol{x}^2 + \boldsymbol{y}^2)^{1/2} \tag{1.3}$$

are C-function for any Euclidean Jordan algebra.

A function that can constitute an equivalent unconstrained minimization problem for the SCCP is called an M-function. In other words, a merit function is a function whose global minima is coincident with the solutions of the original SCCP. For constructing an M-function, the C-function severs an important role.

In order to solve (1.1), we only need to find the solution of the nonlinear equations $\boldsymbol{\phi}(\boldsymbol{x},\ \boldsymbol{F}(\boldsymbol{x})) = 0$ induced via the C-function associated with the symmetric cone. Take FB function as a example, the SCCP is equivalent to a system of nonlinear equations:

$$\boldsymbol{\Phi}(\boldsymbol{x}) = \begin{pmatrix} \boldsymbol{\phi}_{\rm FB}(\boldsymbol{x}_1,\ F(\boldsymbol{x}_1)) \\ \vdots \\ \boldsymbol{\phi}_{\rm FB}(\boldsymbol{x}_n,\ F(\boldsymbol{x}_n)) \end{pmatrix} = 0. \tag{1.4}$$

For each C-function, there is a natural merit function $\boldsymbol{\Psi}_{\rm FB}$ given by

$$\boldsymbol{\Psi}_{\rm FB} := \frac{1}{2}\|\boldsymbol{\Phi}_{\rm FB}\|^2 = \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{\phi}_{\rm FB}(\boldsymbol{x}_i,\ F(\boldsymbol{x}_i))^2, \tag{1.5}$$

from which the SCCP can be recast as an unconstrained minimization

$$\min_{x\in\mathbf{R}^n} \boldsymbol{\Psi}_{\rm FB}(\boldsymbol{x}). \tag{1.6}$$

In this paper, we are particularly interested in the generalized FB, which is presented in a recent paper to deal with NCP by Chen[7]–[8]. The definition of the generalized FB function is as follows.

Let $\boldsymbol{x}, \boldsymbol{y} \in \mathbf{R}^n$. For $p > 1$,

$$\boldsymbol{\phi}_p(\boldsymbol{x},\ \boldsymbol{y}) = \boldsymbol{x} + \boldsymbol{y} - (|\boldsymbol{x}|^p + |\boldsymbol{y}|^p)^{1/p} \tag{1.7}$$

is called the generalized FB function of NCP.

Shortly afterwards, Pan *et al.*[9] developed the M-function method for SOCCP based on the generalized FB function and Kum *et al.*[10] proved that generalized FB function and penalized generalized FB function are complementarity functions for SOCCP. Nowadays, Kum[11] extends the generalized FB function to the SCCP when $p = 1, 2, 3, 4$ and proposes a question that "Is the function a C-function for any positive integer $n \geq 2$?" Motivated by the above mentioned work, we are trying to extend the generalized FB function to the SCCP when $p>1$. Moreover, under suitable conditions, we derive the boundedness of level set of the natural M-function induced by the penalized generalized FB function from a trace inequality in Euclidean Jordan algebras, which is very useful toward an entire development of the M-function theory for SCCP based on the penalized version as a future research.