COMMUNICATIONS
IN
MATHEMATICAL
RESEARCH
32(1)(2016), 47–56

Normality Criteria of Meromorphic Functions Concerning Shared Analytic Function

YANG QI
(*School of Mathematics Science, Xinjiang Normal University, Urumqi, 830054*)

Communicated by Ji You-qing

Abstract: In this paper, we use Pang-Zalcman lemma to investigate the normal family of meromorphic functions concerning shared analytic function, which improves some earlier related results.

Key words: meromorphic function, entire function, shared function, normal family
2010 MR subject classification: 30D35, 30D45
Document code: A
Article ID: 1674-5647(2016)01-0047-10
DOI: 10.13447/j.1674-5647.2016.01.03

1 Introduction and Main Results

Let D be a domain in $\mathbf{C}$, and $\mathcal{F}$ be a family of meromorphic functions defined in the domain D. $\mathcal{F}$ is said to be normal in D if any sequence $\{f_n\} \subset \mathcal{F}$ contains a subsequence f_{n_j}, which converges spherically locally uniformly in D to a meromorphic function or ∞ (see [1]–[5]).

Let $f(z)$ be a mermorphic function in a domain D and $z_0 \in D$. If $f(z_0) = z_0$, we say that z_0 is the fixed-point of $f(z)$. Let $f(z)$ and $g(z)$ denote two meromorphic functions in D. If $f(z) - \psi(z)$ and $g(z) - \psi(z)$ have the same zeros (or ignoring multiplicity), then we say that $f(z)$ and $g(z)$ share $\psi(z)$ CM (or IM).

In 1998, Wang and Fang[6] proved the following result:

Theorem 1.1 *Let k and $n \geq k + 1$ be two positive integers, and f be a transcendental merimorphic function. Then $(f^n)^{(k)}$ assumes every finite nonzero value infinitely often.*

Corresponding to Theorem 1.1, there are the following theorems about normal families.

Theorem 1.2[7] *Let k and $n \geq k + 3$ be two positive integers and $\mathcal{F}$ be a family of meromorphic functions defined in a domain D. If $(f^n)^{(k)} \neq 1$ for every function $f \in \mathcal{F}$, then $\mathcal{F}$ is normal in D.*

Received date: May 27, 2014.
Foundation item: The NSF (11461070) of China.
E-mail address: yangqi_8138@126.com (Yang Q).

In 2009, Li and Gu[8] proved:

Theorem 1.3 *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D. Let $k, n \geq k+2$ be positive integers and $a \neq 0$ be a finite complex number. For each pair $(f, g) \in \mathcal{F}$, if $(f^n)^{(k)}$ and $(g^n)^{(k)}$ share a in D, then $\mathcal{F}$ is normal in D.*

Lately, many authors studied the functions of the form $f(f^{(k)})^n$. Hu and Meng[9] proved:

Theorem 1.4 *Take positive integers n and k with $n, k \geq 2$, and take a non-zero complex number a. Let $\mathcal{F}$ be a family of meromorphic functions in the plane domain D such that each $f \in \mathcal{F}$ has all its zeros of multiplicity at least k. For each pair $(f,\ g) \in \mathcal{F}$, if $f(f^{(k)})^n$ and $g(g^{(k)})^n$ share a IM, then $\mathcal{F}$ is normal in D.*

Recently, Jiang and Gao[10] extended Theorem 1.4 as follows:

Theorem 1.5 *Let $m \geq 0$, $n \geq 2m+2$ and $k \geq 2$ be three positive integers and m be divisible by $n+1$. Suppose that $a(z)(\not\equiv 0)$ is a holomorphic function with zeros of multiplicity m in a domain D. Let $\mathcal{F}$ be a family of meromorphic functions in a domain D, and for each $f \in \mathcal{F}$, f has all its zeros of multiplicity $\max\{k+m,\ 2m+2\}$ at least. For each pair $(f,\ g) \in \mathcal{F}$, if $f(f^{(k)})^n$ and $g(g^{(k)})^n$ share $a(z)$ IM, then $\mathcal{F}$ is normal in D.*

A natural question is: What can be said if the function $f(f^{(k)})^n$ in Theorem 1.5 is replaced by the function $f^d(f^{(k)})^n$? In this paper, we answer this question by proving the following theorem:

Theorem 1.6 *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D, and $m \geq 0$, $n \geq 2m+2$, $k \geq 2$, $d \geq 1$, $p \geq 1$ be five integers and m be divisible by $n+d$. Let $\psi(z) \not\equiv 0$ be an analytic function with zeros of multiplicity m in a domain D. Suppose that every $f \in \mathcal{F}$ has all its zeros of multiplicity at least $p \geq \max\left\{k + \dfrac{m}{d},\ 2m+2\right\}$. For each pair $(f,\ g) \in \mathcal{F}$, if $f^d(f^{(k)})^n$ and $g^d(g^{(k)})^n$ share $\psi(z)$ IM, then $\mathcal{F}$ is normal in D.*

Remark 1.1 Obviously, from Theorem 1.6, we can get Theorem 1.5 when $d = 1$.

2 Some Lemmas

In order to prove Theorem 1.6, we require the following results.

Lemma 2.1[11] *Let $\mathcal{F}$ be a family of meromorphic functions on the unit disc satisfying all zeros of functions in $\mathcal{F}$ have multiplicity $\geq p$ and all poles of functions in $\mathcal{F}$ have multiplicity $\geq q$. Let α be a real number satisfying $-q < \alpha < p$. Then $\mathcal{F}$ is not normal at 0 if and only if there exist*

a) *a number $0 < r < 1$;*
b) *points z_n with $|z_n| < r$;*