Communications in Mathematical Research
32(1)(2016), 57–69

L-octo-algebras

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Communicated by Du Xian-kun

Abstract: L-octo-algebra with 8 operations as the Lie algebraic analogue of octo-algebra such that the sum of 8 operations is a Lie algebra is discussed. Any octo-algebra is an L-octo-algebra. The relationships among L-octo-algebras, L-quadri-algebras, L-dendriform algebras, pre-Lie algebras and Lie algebras are given. The close relationships between L-octo-algebras and some interesting structures like Rota-Baxter operators, classical Yang-Baxter equations and some bilinear forms satisfying certain conditions are given also.

Key words: L-octo-algebra, L-quadri-algebra, bimodule
2010 MR subject classification: 17A30, 17B60
Document code: A
Article ID: 1674-5647(2016)01-0057-13
DOI: 10.13447/j.1674-5647.2016.01.04

1 Introduction

Octo-algebras are a remarkable class of Loday algebras (see [1]). Loday algebras which include dendriform trialgebras (see [2]–[3]), NS-algebras (see [4]), octo-algebras (see [5]), ennea-algebras (see [6]) and dendriform-Nijenhuis algebras (see [4]) were first introduced by Loday[7] in 1995 with motivation from algebraic K-theory. At first, they introduced due to their own interesting motivations, then they developed as independent algebraic systems. Loday algebras are closely related to the study of CYBE, $\mathcal{O}$-operator, operads and so on (see [8]–[10]).

In this paper, we introduce the notion of L-octo-algebra and discuss the relationships among Lie algebra, L-dendriform algebra, L-quadri-algebras and L-octo-algebras. This paper is organized as follows: In Section 2, we recall some basic facts on pre-Lie algebras, L-dendriform algebras and L-quadri-algebras; The definition of L-octo-algebras and the associated L-quadri-algebras, L-dendriform algebras and pre-Lie algebras on L-octo-algebras are given in Section 3; We give the bimodules on L-quadri-algebras and the bimodule of the

Received date: Jan. 26, 2014.
Foundation item: The NSF (11071106 and 11471151) of China.
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associated L-quadri-algebras, L-dendriform algebras, pre-Lie algebras on L-octo-algebras and the construction of L-octo-algebras on L-quadri-algebras by $\mathcal{O}$-operators on L-quadri-algebras, 2-cocycle on L-quadri-algebra in Sections 4 and 5, respectively; Finally, we give the bilinear forms on L-octo-algebras and the LO-equation on L-octo-algebras in Section 6. Throughout this paper, all algebras are finite dimensional and over a field F of characteristic zero.

2 Pre-Lie Algebras, L-dendriform Algebras and L-quadri-algebras

Proposition 2.1[11]–[12] *Let $(A, \circ)$ be a pre-Lie algebra. Then the commutator*

$$[x, y] = x \circ y - y \circ x, \qquad x, y \in A \tag{2.1}$$

defines a Lie algebra $\mathfrak{g}(A)$, which is called the sub-adjacent Lie algebra of A.

Proposition 2.2[11] *Let $(A, \triangleright, \triangleleft)$ be an L-dendriform algebra. If we define*

$$x \bullet y = x \triangleright y + x \triangleleft y, \qquad x, y \in A, \tag{2.2}$$

$$x \circ y = x \triangleright y - y \triangleleft x, \qquad x, y \in A, \tag{2.3}$$

then $(A, \bullet)$ and $(A, \circ)$ are pre-Lie algebras, which are called the associated horizontal and vertical pre-Lie algebras.

Proposition 2.3[12] *Let $(A, \searrow, \nearrow, \nwarrow, \swarrow)$ be an L-quadri-algebra.*

(1) *$(A, \succ, \prec)$ and $(A, \vee, \wedge)$ are dendriform algebras. They are called the associated vertical and depth L-dendriform algebra of $(A, \searrow, \nearrow, \nwarrow, \swarrow)$;*

(2) *If we define*

$$x \triangleright y = x \searrow y - y \nwarrow x, \quad x \triangleleft y = x \nearrow y - y \swarrow x, \qquad x, y \in A, \tag{2.4}$$

then $(A, \triangleright, \triangleleft)$ is a dendriform algebra, which is called the associated horizontal L-dendriform algebra of $(A, \searrow, \nearrow, \nwarrow, \swarrow)$.

3 L-octo-algebras

Definition 3.1[12] *Let A be a vector space with eight bilinear products denoted by $\searrow_1$, $\searrow_2$, $\nearrow_1$, $\nearrow_2$, $\nwarrow_1$, $\nwarrow_2$, $\swarrow_1 \swarrow_2$: $A \otimes A \to A$. $(A, \searrow_1, \searrow_2, \nearrow_1, \nearrow_2, \nwarrow_1, \nwarrow_2, \swarrow_1, \swarrow_2)$ is called an L-octo-algebra if for any $x, y, z \in A$,*

$$x \searrow_2 (y \searrow_2 z) - (x *_{12} y) \searrow_2 z = y \searrow_2 (x \searrow_2 z) - (y *_{12} x) \searrow_2 z,$$

$$x \searrow_2 (y \nearrow_2 z) - (x \vee_{12} y) \nearrow_2 z = y \nearrow_2 (x \succ_2 z) - (y \wedge_{12} x) \nearrow_2 z,$$

$$x \searrow_2 (y \nearrow_1 z) - (x \vee_2 y) \nearrow_1 z = y \nearrow_1 (x \succ_{12} z) - (y \wedge_1 x) \nearrow_1 z,$$

$$x \nearrow_2 (y \succ_1 z) - (x \wedge_2 y) \nearrow_1 z = y \searrow_1 (x \nearrow_{12} z) - (y \vee_1 x) \nearrow_1 z,$$

$$x \searrow_2 (y \searrow_1 z) - (x *_2 y) \searrow_1 z = y \searrow_1 (x \searrow_{12} z) - (y *_1 x) \searrow_1 z,$$

$$x \searrow_2 (y \nwarrow_1 z) - (x \searrow_2 y) \nwarrow_1 z = y \nwarrow_1 (x *_{12} z) - (y \nwarrow_1 x) \nwarrow_1 z,$$