COMMUNICATIONS
IN
MATHEMATICAL
RESEARCH
32(1)(2016), 88–96

Normality Criteria of Meromorphic Functions

WANG QIONG[1], YUAN WEN-JUN[2], CHEN WEI[3]
AND TIAN HONG-GEN[1,*]

(*1. School of Mathematics Science, Xinjiang Normal University, Urumqi, 830054*)

(*2. School of Mathematics and Information Sciences, Guangzhou University, Guangzhou, 510006*)

(*3. School of Mathematics Sciences, Shandong University, Jinan, 250000*)

Communicated by Ji You-qing

Abstract: In this paper, we consider normality criteria for a family of meromorphic functions concerning shared values. Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D, m, n, k and d be four positive integers satisfying $m \geq n+2$ and $d \geq \dfrac{k+1}{m-n-1}$, and $a(\neq 0)$, b be two finite constants. Suppose that every $f \in \mathcal{F}$ has all its zeros and poles of multiplicity at least k and d, respectively. If $(f^n)^{(k)} - af^m$ and $(g^n)^{(k)} - ag^m$ share the value b for every pair of functions $(f,\ g)$ of $\mathcal{F}$, then $\mathcal{F}$ is normal in D. Our results improve the related theorems of Schwick (Schwick W. Normality criteria for families of meromorphic function. *J. Anal. Math.*, 1989, **52**: 241–289), Li and Gu (Li Y T, Gu Y X. On normal families of meromorphic functions. *J. Math. Anal. Appl.*, 2009, **354**: 421–425).

Key words: meromorphic function, shared value, normal criterion

2010 MR subject classification: 30D30, 30D45

Document code: A

Article ID: 1674-5647(2016)01-0088-09

DOI: 10.13447/j.1674-5647.2016.01.07

1 Introduction and Main Results

Let $\mathbf{C}$ be the set of complex numbers, D be a domain in $\mathbf{C}$, which means that D is a connected nonempty open subset of $\mathbf{C}$. Let $\mathcal{F}$ be a family of meromorphic functions defined in D. For $\{f,\ g\} \subset \mathcal{F}$, $\{a,\ b\} \subset \mathbb{P}^1 = \mathbf{C} \cup \{\infty\}$, we write $f = a \Rightarrow g = b$ ($f = a \Leftrightarrow g = b$) if $f^{-1}(a) \subset g^{-1}(b)$ ($f^{-1}(a) = g^{-1}(b)$), and say that f and g share a ignoring multiplicities

Received date: Jan. 6. 2015.

Foundation item: The NSF (11461070, 11271090) of China, the NSF (S2012010010121, 2015A030313346) of Guangdong Province, and the Graduate Research, and Innovation Projects (XJGRI2015106) of Xinjiang Province.

* **Corresponding author**.

E-mail address: wq1298592600@163.com (Wang Q), tianhg@xjnu.edu.cn (Tian H G).

(IM, for short) if $f^{-1}(a) = g^{-1}(a)$ (see [1]). Here, the family $\mathcal{F}$ is said to be normal in D if any sequence of $\mathcal{F}$ must contain a subsequence that locally uniformly spherically converges to a meromorphic function or ∞ in D (see [2]).

In 1989, Schwick[3] proved a normality criterion:

Theorem 1.1 *Let k, $n(\geq k+3)$ be two positive integers, and $\mathcal{F}$ be a family of meromorphic functions defined in a domain D. If $(f^n)^{(k)} \neq 1$ for every function $f \in \mathcal{F}$, then $\mathcal{F}$ is normal in D.*

In 1998, Wang and Fang[4] proved:

Theorem 1.2 *Let k, $n(\geq k+1)$ be two positive integers, and f be a transcendental meromorphic function. Then $(f^n)^{(k)}$ assumes every finite non-zero value infinitely often.*

For families of meromorphic functions, the connection between normality and shared values has been studied frequently.

By the ideas of shared values, Li and Gu[5] proved the following results:

Theorem 1.3 *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D, k, $n(\geq k+2)$ be two positive integers, and $a \neq 0$ be a finite complex number. If $(f^n)^{(k)}$ and $(g^n)^{(k)}$ share a in D for every pair of functions f, $g \in \mathcal{F}$, then $\mathcal{F}$ is normal in D.*

In 2011, Liu and Li[6] studied Theorem 1.3, in which the value a was replaced by the fix-point z, and got the following result:

Theorem 1.4 *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D, k, $n(\geq k+1)$ be two positive integers. If $(f^n)^{(k)}$ and $(g^n)^{(k)}$ share z in D for every pair of functions f, $g \in \mathcal{F}$, then $\mathcal{F}$ is normal in D.*

Lately, some theorems in this area appear. Hu and Meng[7], Jiang and Gao[8] studied the functions of the form $f(f^{(k)})^n$. Ding *et al.*[9] studied the functions of the form $f^m(f^{(k)})^n$ and Sun[10] studied the form $P(f)(f^{(k)})^m$.

Naturally, we pose the following question:

Question Whether the form $(f^n)^{(k)} - af^m$ in above Theorems can have similar results?

In this paper, we prove the following theorems and deal with this question.

Theorem 1.5 *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain D, m, n, k be three positive integers satisfying $m \geq n+k+3$, and $a(\neq 0)$, b be two finite complex constants. If $(f^n)^{(k)} - af^m \neq b$ for every functions f of $\mathcal{F}$, then $\mathcal{F}$ is normal in D.*

Whether the condition $m \geq n+k+3$ in Theorem 1.5 can be improved? We get the following results: