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A Formula for Khovanov Type Link Homology of Pretzel Knots

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Abstract: Khovanov type homology is a generalization of Khovanov homology. The main result of this paper is to give a recursive formula for Khovanov type homology of pretzel knots $P(-n, -m, m)$. The computations reveal that the rank of the homology of pretzel knots is an invariant of n. The proof is based on a "shortcut" and two lemmas that recursively reduce the computational complexity of Khovanov type homology.

Key words: pretzel knot, Khovanov type homology, Frobenius algebra, TQFT

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1 Introduction

Khovanov homology[1] is a modern and powerful invariant for knots and links related to the representation theory, physics, and symplectic geometry. As a graded homology theory, it is a categorification of the Jones polynomial. In [2], a new homology theory of knots and links over a ring R, Khovanov type homology, is constructed from a Frobenius algebra, and its corresponding geometric interpretation is obtained. The link homology theory is different from the known natural generalizations (see [3]–[7]).

It is well known that the 3-strand pretzel knots are well-studied sources of examples in the knot theory. Khaled[8] proved a recursion formula for the rational Khovanov homology

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of families of pretzel links $P(p,r,r)$. Suzuki[9] calculated the rational Khovanov homology of a class of pretzel knots $P(p,q,r)$ by using the spectral sequence constructed by Turner[10]. Manion[11] gave a formula for the unreduced Khovanov homology over the rational numbers of all 3-strand pretzel links. However, computing the generalized Khovanov homology over R remains mysterious in many cases.

In this paper, we set out to calculate Khovanov type homology of infinite classes of knots, pretzel knots $P(-n,-m,m)$. We utilize the simplicity of the diagrams resulting from resolving one of the crossings to make an inductive argument in terms of n. Thus, we provide a recursive formula for Khovanov type homology of pretzel knots.

The organization of the coming sections is as follows. In Section 2, we give a brief summary of Khovanov type homology and review a computational shortcut to Khovanov homology. Similarly, we can use the "shortcut" to calculate Khovanov type homology. In Section 3, after proving two lemmas of Khovanov type homology, we obtain a formula for pretzel knots $P(-n,-m,m)$.

2 Preliminary

2.1 Khovanov Type Homology

For a ring R, a (1+1) topological quantum field theory (TQFT for short, see [12]–[13]) which is a functor on (1+1) cobordisms (see [5]) valued in the category R-module has been studied in [1].

The authors firstly construct a Frobenius algebra (see [14]), which is a free R-module over the basic ring of rank 2 generated by 1 and x.

To get a algebra structure, a multiplication $m : A\otimes A\to A$ is given by

$$m(1\otimes 1)=1,\qquad m(1\otimes x)=m(x\otimes 1)=x,\qquad m(x\otimes x)=hx.$$

For simplification, we abbreviate $x\otimes x=hx$ by $x^2=hx$, where $h\in R$ and $h^{n-1}=0$ which implies that $x^n=0$ for $n\geq 3$.

To get a coalgebra structure, a comultiplication $\Delta : A\to A\otimes A$ is given by

$$\Delta(1)=1\otimes x+x\otimes 1-h1\otimes 1,\qquad \Delta(x)=x\otimes x.$$

We introduce a $\mathbb{Z}$-grading on R by $\deg(h)=-2$, $\deg(1)=1$, $\deg(x)=-1$.

Consequently, given a link diagram D, the cube of resolutions is built from it, where vertices of the cube are of all possible 0 and 1 resolutions of the crossings. The resolutions are represented by the Kauffman states (see [15]) themselves as shown in Fig. 2.1. The edges of the cube are certain given surfaces to which the appropriate signs are attached. The entire cube is "summed" into a chain complex, $\bar{C}(D)$, in the appropriate geometric category.

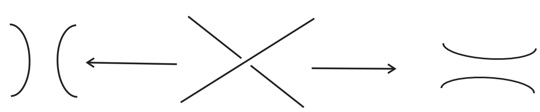

Fig. 2.1 The resolution 0 and 1 of a crossing