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Stochastic Maximum Principle for Optimal Control of Forward-backward Stochastic Pantograph Systems with Regime Switching

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Abstract: In this paper, we derive the stochastic maximum principle for optimal control problems of the forward-backward Markovian regime-switching system. The control system is described by an anticipated forward-backward stochastic pantograph equation and modulated by a continuous-time finite-state Markov chain. By virtue of classical variational approach, duality method, and convex analysis, we obtain a stochastic maximum principle for the optimal control.

Key words: stochastic control, stochastic maximum principle, anticipated forward-backward stochastic pantograph equation, variational approach, regime switching, Markov chain

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1 Introduction

It is well known that stochastic optimal control problem plays an important role in control theory. Since 1970's, many scholars committed to obtaining the maximum principle for the stochastic control system such as Kushner[1], Bismut[2]–[4], Bensoussan[5]–[6], etc., and acquired very important achievement. The general stochastic maximum principle was obtained by Peng[7] by introducing the second order adjoint equations, which allowed the control enter in both the drift and diffusion coefficients while the control domain was non-convex. This groundbreaking work makes the stochastic optimal control problem has been

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vigorous development.

Stochastic delay differential equations (SDDEs) have come to play an important role in many branches of science and industry. Such models have been used with great success in a variety of application areas, including biology, epidemiology, mechanics, economics and finance. Recently, as a special case of SDDEs, the following stochastic pantograph equations (SPEs) has been received a great deal of attention (see [8]).

$$\begin{cases} \mathrm{d}x(t) = b(t,\ x(t),\ x(qt))\mathrm{d}t + \sigma(t,\ x(t),\ x(qt))\mathrm{d}B(t), \qquad t \in [0, T], \\ x(0) = x_0, \qquad q \in (0, 1). \end{cases}$$

Bismut[2] introduced the linear backward stochastic differential equations (BSDEs). Pardoux and Peng[9] proved the existence and uniqueness for the solution of the nonlinear BSDEs. The applications of a regime-switching model in finance and stochastic control have received significant attention in recent years. The regime-switching model in economic and finance fields was first introduced by Hamilton[10] to describe a time series model and then intensively investigated in the past two decades in mathematical finance. Dokuchaev and Zhou[11] studied a kind of maximum principle when the system dynamics were controlled BSDEs. Then, the forward-backward maximum principle was generalized and applied in finance. We introduce a new type of BSDE, called the anticipated BSDE with Markov chains, as our adjoint equation as follows:

$$\begin{cases} -\mathrm{d}Y_t = g(t,\ Y_t,\ Y_{qt},\ Z_t,\ Z_{qt},\ V_t)\mathrm{d}t - Z_t\mathrm{d}B_t - \sum\limits_{j\in\mathbb{I}} V_t(j)\mathrm{d}\tilde{\mathcal{V}}_t(j), \qquad t \in [0, T], \\ Y_t = a_t,\ Z_t = b_t, \qquad q \in (0, 1),\ t \in \left[T,\ \dfrac{T}{q}\right]. \end{cases}$$

In this paper, by using the results about BSDEs with Markov chains in [12]–[14], we derive maximum principle for the forward-backward regimes-witching model. To the authors' knowledge, it is the first time to investigate this system.

We sketch out the organization of the paper. In Section 2, we give the preliminaries about anticipated BSDEs with Markov chain and formulates our optimal control problems. We derive the stochastic maximum principle for the optimal control by virtue of the duality method and convex analysis in Section 3.

2 Preliminaries and Formulation of the Optimal Control Problems

2.1 Preliminaries

Let $(\varOmega,\ \mathcal{F},\ \mathcal{P})$ be a probability space. $T > 0$ is a finite-time horizon. $\{B_t\}_{0\le t\le T}$ is a d-dimensional Brownian motion and $\{\alpha_t\}_{0\le t\le T}$ is a finite-state Markov chain with the state space given by $\mathbb{I} = \{1, 2, \cdots, k\}$. Assume that B and α are independent. The transition intensities are $\lambda(i, j)$ for $i \neq j$ with $\lambda(i, j)$ non-negative, uniformly bounded, and $\lambda(i, j) = -\sum\limits_{i\neq j} \lambda(i, j)$. Let $\{\mathcal{F}_t\}_{0\le t\le T}$ be the filtration generated by $\{B_t,\ \alpha_t\}_{0\le t\le T}$ and augmented by all P-null sets of $\mathcal{F}$.