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Extended tanh-function Method for Solving Traveling Wave Solutions of Nonlinear Kundu Equation

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Abstract: In this paper, by using the balancing method and the extended tanh-function method, we obtain the exact traveling wave solutions of Kundu equation with fifth-order nonlinear term. Applications of this method to some other nonlinear partial differential equations are also presented.

Key words: extended tanh-function method, Kundu equation, traveling wave solution

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1 Introduction

It is well known that complex physical phenomena are described by nonlinear partial differential equations. The solutions of these equations have important significance in mathematical physics and engineering. Therefore, investigating traveling wave solutions is becoming significant. In the past several decades, various methods have been proposed such as the inverse scattering method (see [1]), Darboux transformation (see [2]), the homogeneous balance method (see [3]), Jacobi elliptic function expansion method (see [4]), the (G'/G)-expansion

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method (see [5]–[6]), the tanh-function method (see [7]) and the extended tanh-function method (see [8]).

Consider the Kundu equation with fifth-order nonlinear term

$$\mathrm{i}u_t + u_{xx} + c_3|u|^2u + c_5|u|^4u - \mathrm{i}s_2(|u|^2u)_x - \mathrm{i}r(|u|^2)_x u = 0, \qquad x \in \mathbf{R}, \tag{1.1}$$

where c_3, c_5, s_2 and r are real numbers. For convenience we rewrite the equation (1.1) as follows

$$u_t = \mathrm{i}u_{xx} + \mathrm{i}(c_3|u|^2 + c_5|u|^4)u + \alpha|u|^2u_x + \beta|u|^2\bar{u}_x, \qquad x \in \mathbf{R}, \tag{1.2}$$

where $\alpha = 2s_2 + r$ and $\beta = s_2 + r$. It is easy to see that the equations (1.1) and (1.2) contain some special cases as follows:

(i) When $c_3 = c_5 = s_2 = 0$, the equation (1.1) becomes the nonlinear Schrödinger equation, i.e.,

$$\mathrm{i}u_t + u_{xx} - \mathrm{i}r(|u|^2)_x u = 0, \qquad x \in \mathbf{R}.$$

(ii) When $c_3 = c_5 = 0$, $s_2 \neq 0$, the equation (1.1) becomes the nonlinear Schrödinger equation with fifth-order term, i.e.,

$$\mathrm{i}u_t + u_{xx} - \mathrm{i}s_2(|u|^2u)_x - \mathrm{i}r(|u|^2)_x u = 0, \qquad x \in \mathbf{R}.$$

(iii) When $c_3 = c_5 = 0$, $s_2 = -\delta$ and $r = -s_2$, the equation (1.2) becomes the Chen-Lee-Liu equation, i.e.,

$$\mathrm{i}u_t + u_{xx} + \mathrm{i}\delta|u|^2u_x = 0, \qquad x \in \mathbf{R}.$$

(iv) When $c_5 = 2\delta^2$, $s_2 = 2\delta$ and $r = -2s_2$, the equation (1.2) becomes a Gerdjikov-Ivanov equation, i.e.,

$$\mathrm{i}u_t + u_{xx} + c_3|u|^2u + 2\delta^2|u|^4u + 2\mathrm{i}\delta u^2\bar{u}_x = 0, \qquad x \in \mathbf{R}.$$

Hence it is significant to study the exact solutions of the Kundu equation. The exact traveling wave solutions of it have been investigated by some authors. Biawas[9] studied the generalized Kundu equation and obtained a solitary solution of it by integration. In [10], the authors obtained the exact solitary waves of Kundu equation by using proper transformations and coefficient method. In [11], the author obtained a new exact solitary wave solution of Kundu equation based on auxiliary equation method and the method of an auxiliary equation of triangle function type with function transformation. The aim of this paper is to explore new exact traveling wave solutions for (1.1) by the extended tanh-function method.

2 Extended tanh-function Method

Consider a nonlinear evolution equation

$$F(u, u_x, u_t, u_{xx}, u_{t,x}, u_{tt}, \cdots) = 0. \tag{2.1}$$

Firstly, we suppose that the traveling solutions of (2.1) with the form $u(t,x) = v(\eta)$, $\eta = x + ct$ or $\eta = x - ct$, where c is wave velocity. Substituting these into (2.1) yields the following ordinary differential equation

$$H(v, v_\eta, v_{\eta\eta}, v_{\eta\eta\eta}, \cdots) = 0. \tag{2.2}$$