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On Reducibility of Beam Equation with Quasi-periodic Forcing Potential

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Abstract: In this paper, the Dirichlet boundary value problems of the nonlinear beam equation $u_{tt} + \Delta^2 u + \alpha u + \epsilon\phi(t)(u + u^3) = 0$, $\alpha > 0$ in the dimension one is considered, where $u(t, x)$ and $\phi(t)$ are analytic quasi-periodic functions in t, and ϵ is a small positive real-number parameter. It is proved that the above equation admits a small-amplitude quasi-periodic solution. The proof is based on an infinite dimensional KAM iteration procedure.

Key words: beam equation, infinite dimension, Hamiltonian system, KAM theory, reducibility

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1 Introduction and Main Result

The well-known Floquet theorem of ordinary differential equations states that any time-periodic linear equation $\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x}$, where $\boldsymbol{x}$ is n-dimensional real or complex vector and $\boldsymbol{A}(t)$ is a continuous $n \times n$ periodic matrix, by means of a periodic change of variables $\dot{\boldsymbol{x}} = \boldsymbol{P}(t)\boldsymbol{y}$, the above system can be reduced to constant coefficients system $\dot{\boldsymbol{y}} = \boldsymbol{B}\boldsymbol{y}$, where $\boldsymbol{P}(t)$ is a periodic matrix (with the same period as $\boldsymbol{A}(t)$). But when $\boldsymbol{A}(t)$ depends on time quasi-periodically, the above system is not always reducible (see [1] for example). In this case, Johnson and Sell[1] gave sufficient conditions which guarantee the reducibility, and Cappel[2] gave sufficient and necessary condition for almost reducibility. But their method fails when pure imaginary spectrum appears.

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Bogolyubov[3] proved reducibility of non autonomous finite-dimensional linear systems to constant coefficient equations by KAM's method. Since then, the reducibility of finite-dimensional system can be dealt by means of the KAM tools. Then, for infinite dimensional system in [4], Dinaburg and Sinai proved that the linear Schrödinger equation

$$\ddot{x} + q(\omega_1 t, \cdots, \omega_r t)x = \lambda x$$

is reducible for "most" large enough λ, when ω is fixed and satisfies the Diophantine conditions

$$|\langle k, \omega\rangle| > \frac{\gamma^{-1}}{|k|^\sigma}, \qquad 0 \neq k \in \mathbf{Z}^r,$$

where $\gamma > 1$, $\sigma > r - 1$ are fixed positive constants. The latest result is in [5], Eliasson and Kuksin proved that the linear d-dimensional Schrödinger equation

$$\dot{u} = -\mathrm{i}(\Delta u - \epsilon V(\varphi_0 + t\boldsymbol{\omega}),\ x;\ \boldsymbol{\omega})u$$

about x-periodic and t-quasi-periodic, which can be reduced to an autonomous equation for most values of the frequency vector $\boldsymbol{\omega}$ by KAM theory.

In this paper, we prove that the non-autonomous nonlinear beam equation admits small-amplitude quasi-periodic solution. The important point is to reduce the beam equation to the system which can be used by KAM theory.

Consider the following nonlinear beam equation with Dirichlet boundary conditions:

$$u_{tt} = -(\Delta^2 u + \alpha u + \epsilon\phi(t)(u + u^3)), \qquad \alpha > 0, \tag{1.1}$$

$$u(t, 0) = u(t, \pi) = u_{tt}(t, 0) = u_{tt}(t, \pi), \qquad -\infty < t < \infty, \tag{1.2}$$

where ϵ is a small positive parameter, $\phi(t)$ is a real analytic quasi-periodic function in t with frequency vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_d) \subset \mathbf{R}^d$. Considering the non-autonomous equation (1.1) in the complex Hilbert space $\ell^{a,\rho}$, first, we choose any fixed lattice points

$$J = \{i_j \mid i_j < i_h,\ 0 < j < h \leq n\},$$

without loss of generality, we suppose $J = \{1, 2, \cdots, n\}$.

Let $A = -\Delta^2 + \alpha$. The eigenvalues and eigenfunctions of the operator A with Dirichlet boundary conditions are as follows

$$\lambda_j^2 = j^4 + \alpha, \quad \phi_j(x) = \sqrt{\frac{1}{\pi}}\sin(jx), \qquad j = 1, 2, \cdots \tag{1.3}$$

When $\epsilon = 0$ in (1.1), the solution of the linear equation

$$u_{tt} = -(\Delta^2 u + \alpha u)$$

with Dirichlet boundary conditions are given by

$$u(t, x) = \sum_{j\geq 1} q_j(t)\phi_j(x).$$

Let us state our main result as follows.

Theorem 1.1 *Consider the* 1*D nonlinear beam equation* (1.1) *with Dirichlet boundary condition* (1.2), *where* $\phi(t)$ *is quasi-period function with* t. *For each index* $J = \{j_1 < j_2 < \cdots < j_n\}$ *and all* $\alpha > 0$, *there exists a Cantor set* $\mathcal{O}_* \subset \mathcal{O}$ *such that* $\xi \in \mathcal{O}_*$. *Then the boundary value problem* (1.1)–(1.2) *has a linearly stable quasi-period solution.*