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Vertex-distinguishing E-total Coloring of Complete Bipartite Graph $K_{7,n}$ when $7 \leq n \leq 95$

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Abstract: Let G be a simple graph. A total coloring f of G is called an E-total coloring if no two adjacent vertices of G receive the same color, and no edge of G receives the same color as one of its endpoints. For an E-total coloring f of a graph G and any vertex x of G, let $C(x)$ denote the set of colors of vertex x and of the edges incident with x, we call $C(x)$ the color set of x. If $C(u) \neq C(v)$ for any two different vertices u and v of $V(G)$, then we say that f is a vertex-distinguishing E-total coloring of G or a VDET coloring of G for short. The minimum number of colors required for a VDET coloring of G is denoted by $\chi^e_{vt}(G)$ and is called the VDET chromatic number of G. The VDET coloring of complete bipartite graph $K_{7,n}$ $(7 \leq n \leq 95)$ is discussed in this paper and the VDET chromatic number of $K_{7,n}$ $(7 \leq n \leq 95)$ has been obtained.

Key words: graph, complete bipartite graph, E-total coloring, vertex-distinguishing E-total coloring, vertex-distinguishing E-total chromatic number
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1 Introduction and Notations

Graph theory is the historical foundation of the science of networks and the basis of information science. The problem in which we are interested is a particular case of the great variety of different ways of labeling a graph.

For an edge coloring (proper or not) g of G and a vertex x of G, let $S(x)$ be the set (not multiset) of colors of the edges incident with x under g.

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For a proper edge coloring, if $S(u) \neq S(v)$ for any two distinct vertices u and v, then the coloring is called a vertex-distinguishing proper edge coloring. The minimum number of colors required for a vertex-distinguishing proper edge coloring of G is denoted by $\chi'_s(G)$. This coloring is proposed in [1] and [2] independently. Many scholars have studied this parameter in [1]–[7].

For an edge coloring which is not necessarily proper, if $S(u) \neq S(v)$ for any two distinct vertices u and v, then the coloring is called a point distinguishing edge coloring. The minimum number of colors required for a point distinguishing edge coloring of G is denoted by $\chi_0(G)$. This coloring is proposed by Harary *et al.*[8] This parameter has been researched in many papers (see [9]–[14]).

For a total coloring (proper or not) f of G and a vertex x of G, let $C(x)$ be the set (not multiset) of colors of vertex x and edges incident with x under f.

For a proper total coloring, if $C(u) \neq C(v)$ for any two distinct vertices u and v, then the coloring is called a vertex-distinguishing (proper) total coloring, or a VDT coloring of G for short. The minimum number of colors required for a VDT coloring of G is denoted by $\chi_{vt}(G)$.

The vertex-distinguishing proper total colorings of graphs are introduced and studied by Zhang *et al.*[15]. After studying the vertex-distinguishing proper total coloring of complete graph, star, complete bipartite graph, wheel, fan, path and cycle, a conjecture was proposed by Zhang *et al.*[15]: Let $\mu(G) = \min\left\{k : \binom{k}{i+1} \geq n_i,\ \delta \leq i \leq \Delta\right\}$, then $\chi_{vt}(G) = \mu(G)$ or $\mu(G) + 1$. In [16], the vertex-distinguishing total coloring of n-cube were discussed, respectively. In [17], the relations of vertex-distinguishing total chromatic numbers between a subgraph and its supergraph had been studied.

In the following we consider a kind of not necessarily proper total coloring which is vertex-distinguishing. A total coloring f of G is called an E-total coloring if no two adjacent vertices of G receive the same color, and no edge of G receives the same color as one of its endpoints. If f is an E-total coloring of graph G and for any $u, v \in V(G)$, $u \neq v$, we have $C(u) \neq C(v)$, then f is called a vertex-distinguishing E-total coloring, or a VDET coloring briefly. The minimum number of colors required for a VDET coloring of G is called the vertex-distinguishing E-total chromatic number of G and is denoted by $\chi^e_{vt}(G)$.

The VDET colorings of complete graph, complete bipartite graph $K_{2,n}$, star, wheel, fan, path and cycle were discussed by Chen *et al.*[18]. A parameter was introduced in [18]:

$$\eta(G) = \min\left\{l : \binom{l}{2} + \binom{l}{3} + \cdots + \binom{l}{i+1} \geq n_\delta + n_{\delta+1} + \cdots + n_i,\ 1 \leq \delta \leq i \leq \Delta\right\},$$

where G is a graph with no isolated vertex and n_i denotes the number of vertices with degree i, $\delta \leq i \leq \Delta$. At the end of the paper [18], a conjecture was proposed.

Conjecture 1.1[18] *For a graph G with no isolated vertices and chromatic number at most* 5, *we have $\chi^e_{vt}(G) = \eta(G)$ or $\eta(G) + 1$.*

We have studied the vertex-distinguishing E-total colorings of mC_3 and mC_4 in [19] and confirmed Conjecture 1.1 for these two kinds of graphs.