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C^2 Continuous Quartic Hermite Spline Curves with Shape Parameters

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Abstract: In order to relieve the deficiency of the usual cubic Hermite spline curves, the quartic Hermite spline curves with shape parameters is further studied in this work. The interpolation error and estimator of the quartic Hermite spline curves are given. And the characteristics of the quartic Hermite spline curves are discussed. The quartic Hermite spline curves not only have the same interpolation and continuity properties of the usual cubic Hermite spline curves, but also can achieve local or global shape adjustment and C^2 continuity by the shape parameters when the interpolation conditions are fixed.

Key words: Hermite spline curve, interpolation curve, shape adjustment, C^2 continuous

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1 Introduction

With the development of geometric design industry, the shapes of curves often need to be changed freely. Hence, the curves with shape parameters have been paid more and more attention by many researchers in geometric modeling. Such as the Bézier curves with shape parameters (see [1]–[3]), the B-spline curves with shape parameters (see [4]–[6]), the trigonometric spline curves with shape parameters (see [7]–[9]), and so on. Those curves with shape parameters not only inherit the similar or same properties of the corresponding usual curves, but also have better performance ability because of the shape parameters.

As a class of interpolation curves, the cubic Hermite spline curves have been widely

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used in many fields. However, when the interpolation conditions are fixed, the usual cubic Hermite spline curves can only be C^1 continuous and theirs shape cannot be changed. Hence, the deficiency of the usual cubic Hermite spline curves limits their applications in practical engineering. For relieving the deficiency of the usual cubic Hermite spline curves, construction of the Hermite spline curves with shape parameters would be an effective way. Li *et al.*[10] presented a class of quartic Hermite spline curves with parameters, which could achieve shape adjustment and C^2 continuity by the parameters. When using the Hermite spline presented by Li *et al.*[10] to construct C^2 curves, the initial value of the parameter should be set in advance and the other parameters are calculated by the recursion formula. Although this method is simple and convenient, its disadvantages lie in that the initial value of the parameter need to be given artificially. When the initial value of the parameter is not appropriate, the curves would be not ideal. In addition, Li *et al.*[10] did not give the interpolation error and its estimator of the quartic Hermite spline curves. For these reasons, the main purpose of this work is to further discuss the quartic Hermite spline curves with shape parameters presented by Li *et al.*[10]. The relationship between the initial parameter and the other parameters is derived, and then the minimum approximation error is used to determine the value of initial parameter, so that selection of the parameters has stronger maneuverability. Besides, the interpolation error and estimator of the quartic Hermite spline curves are given.

The rest of this paper is organized as follows. In Section 2, the usual cubic Hermite spline curves are discussed. In Section 3, the quartic Hermite basis functions with shape parameters are defined, and the properties of the basis functions are given. In Section 4, definition and properties of the quartic Hermite spline curves with shape parameters are given. In Section 5, characteristics of the quartic Hermite spline curves are studied. A short conclusion is given in Section 6.

2 The Usual Cubic Hermite Spline Curves

Given a function $y = r(x)$ $(a \leq x \leq b)$, Δ: $a = x_0 < x_1 < \cdots < x_n = b$ is a division of $[a, b]$, $y_i = r(x_i)$, $d_i = r'(x_i)$, set $h_i = x_{i+1} - x_i$, $t = \dfrac{x - x_i}{h_i}$, the corresponding usual cubic Hermite spline curves can be expressed as follows (see [11]):

$$G_i(x) = \alpha_0(t)y_i + \alpha_1(t)y_{i+1} + \beta_0(t)h_i d_i + \beta_1(t)h_i d_{i+1}, \tag{2.1}$$

where $x \in [x_i, x_{i+1}]$ $(i = 0, 1, 2, \cdots, n-1)$, $\alpha_j(t)$ and $\beta_j(t)$ $(j = 0, 1)$ are the usual cubic Hermite basis functions defined as follows:

$$\begin{cases} \alpha_0(t) = 1 - 3t^2 + 2t^3, \\ \alpha_1(t) = 3t^2 - 2t^3, \\ \beta_0(t) = t(1-t)^2, \\ \beta_1(t) = -t^2(1-t). \end{cases} \tag{2.2}$$

By a simple deduction, the usual cubic Hermite basis functions expressed as (2.2) satisfy