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Multilinear Calderón-Zygmund Operators and Their Commutators with BMO Functions in Herz-Morrey Spaces with Variable Smoothness and Integrability

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Abstract: In this paper, we obtain that multilinear Calderón-Zygmund operators and their commutators with BMO functions are bounded on products of Herz-Morrey spaces with variable smoothness and integrability. The vector-valued setting of multilinear Calderón-Zygmund operators is also considered.

Key words: multilinear Calderón-Zygmund operator, variable exponent, Herz-Morrey space, vector valued estimate
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1 Introduction

Recent decades, variable exponent function spaces have been received more and more attention. This mainly began with the work of Kováčik and Rákosník in 1991. In [1], Kováčik and Rákosník gave fundamental properties of the variable Lebesgue and Sobolev spaces. Then some sufficient conditions were obtained for the boundedness of Hardy-Littlewood maximal operator on variable Lebesgue spaces (see [2]). After that, many function spaces with variable exponents appeared, such as Besov and Trieble-Lizorkin spaces with variable exponents, Hardy spaces with variable exponents, Morrey spaces with variable exponent, Bessel potential spaces with a variable exponent and Herz-Morrey spaces with variable exponents (see [3]–[17]). Herz-Morrey spaces are a generalization of Herz spaces. For the classical Herz spaces we refer the reader to the monograph by Lu *et al.*[18]

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Recently, linear and multilinear singular operators and their commutators are also intensively studied by a significant number of authors, for instance, see [19]–[25].

In this paper we consider the boundedness of multilinear Calderón-Zygmund singular operators, their commutators with BMO functions and their vector-valued setting in Herz-Morrey spaces with three variable exponents $M\dot{K}^{\alpha(\cdot),\lambda}_{q(\cdot),p(\cdot)}(\mathbf{R}^n)$.

2 Main Results

To state the main results of this paper, we need recall some notions firstly.

Let m be an integer not less than 2. A multilinear operator T is called a Calderón-Zygmund operator if it is initially defined on the m-fold product of the Schwartz space $\mathcal{S}(\mathbf{R}^n)$ and can be extended bounded from $L^{p_1}\times L^{p_2}\times\cdots\times L^{p_m}$ to L^p for some $1<p_1$, $\cdots$, $p_m<\infty$ with $\dfrac{1}{p_1}+\dfrac{1}{p_2}+\cdots+\dfrac{1}{p_m}=\dfrac{1}{p}$, and for $f_1,\ \cdots,\ f_m\in L^\infty_{\rm C}(\mathbf{R}^n)$ (the space of compactly supported bounded functions), $x\notin\bigcap\limits_{j=1}^{m}\operatorname{supp} f_j$, $\boldsymbol{f}=(f_1,\ \cdots,\ f_m)$,

$$T\boldsymbol{f}(x):=\int_{(\mathbf{R}^n)^m}K(x,y_1,\cdots,y_m)\prod_{i=1}^m f_i(y_i)\mathrm{d}y_1\cdots\mathrm{d}y_m,$$

where the kernel K is a function in $(\mathbf{R}^n)^{m+1}$ away from the diagonal $y_0=y_1=\cdots=y_m$ and there exist positive constants ϵ, A such that

$$|K(x,y_1,\cdots,y_m)|\le A\left(\sum_{i=1}^m|x-y_i|\right)^{-mn},$$

$$|K(x,y_1,\cdots,y_m)-K(x',y_1,\cdots,y_m)|\le\frac{A|x-x'|^\epsilon}{\left(\sum\limits_{i=1}^m|x-y_i|\right)^{mn+\epsilon}}$$

provided that

$$|x-x'|\le\frac{1}{2}\max\{|x-y_1|,\cdots,|x-y_m|\},$$

and for each $i\in\{1,2,\cdots,m\}$,

$$|K(x,y_1,\cdots,y_i,\cdots,y_m)-K(x,y_1,\cdots,y_i',\cdots,y_m)|\le\frac{A|y_i-y_i'|^\epsilon}{\left(\sum\limits_{j=1}^m|x-y_j|\right)^{mn+\epsilon}}$$

provided that

$$|y_i-y_i'|\le\frac{1}{2}\max\{|x-y_1|,\cdots,|x-y_m|\}.$$

Grafakos and Torres[26] showed that if T is an m-linear Calderón-Zygmund operator, then T is bounded from $L^{q_1}\times L^{q_2}\times\cdots\times L^{q_m}$ to L^q for each $1<q_1,q_2,\cdots,q_m<\infty$ such that $\dfrac{1}{q_1}+\dfrac{1}{q_2}+\cdots+\dfrac{1}{q_m}=\dfrac{1}{q}$. Moreover, Grafakos and Torres[19] obtained weighted norm inequalities for multilinear Calderón-Zygmund operators.