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The New Structure Theorem of Right-e Wlpp Semigroups

WANG CHUN-RU[1], REN XUE-MING[2] AND MA SI-YAO[2]

(*1. Huaqing College, Xi'an University of Architecture and Technology, Xi'an, 710043*)

(*2. Department of Mathematics, Xi'an University of Architecture and Technology, Xi'an, 710055*)

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Abstract: Wlpp semigroups are generalizations of lpp semigroups and regular semigroups. In this paper, we consider some kinds of wlpp semigroups, namely right-e wlpp semigroups. It is proved that such a semigroup S, if and only if S is the strong semilattice of $\mathcal{L}$-right cancellative planks; also if and only if S is a spined product of a right-e wlpp semigroup and a left normal band.

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1 Introduction

According to Tang[1], a new generalized Green relation $\mathcal{R}^{**}$ on a semigroup S is defined as follows: for any $a, b \in S$,

$$(a,\, b) \in \mathcal{R}^{**} \Longleftrightarrow [\forall x, y \in S^1,\ (xa,\, ya) \in \mathcal{L} \leftrightarrow (xb,\, yb) \in \mathcal{L}],$$

where $\mathcal{L}$ is the usual Green relation. It is easy to verify that $\mathcal{R}^{**}$ is a left congruence on any semigroup and $\mathcal{R} \subseteq \mathcal{R}^{**}$. A semigroup S is said to a wlpp semigroup if each class $\mathcal{R}^{**}$ of S contains an idempotent of S and $a = ea$ for any $a \in S$ and $e \in R_a^{**} \cap E(S)$, where R_a^{**} is the $\mathcal{R}^{**}$-class of S containing the element a. It is easy to check that a regular semigroup S is a wlpp semigroup and a wlpp semigroup is a generalization of a regular semigroup. In this paper, we consider the following semigroups:

Definition 1.1[2] *A wlpp semigroup S is called a right-e wlpp semigroup if $xey = xye$ holds for any $e \in E(S)$ and any $x, y \in S^1$ with $x \neq 1$.*

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E-mail address: chunru123@163.com (Wang C R).

We first have the following result for right-e wlpp semigroups which will be frequently used in the sequel.

Lemma 1.1 *If S is a right-e wlpp semigroup, then every $\mathcal{R}^{**}$-class of S contains a unique idempotent.*

Proof. Suppose that S is a right-e wlpp semigroup and $a \in S$. Then there exists an idempotent $e \in R_a^{**} \cap E(S)$ such that $a = ea$. Hence,

$$ae = eae = eea = ea = a.$$

Now if $f \in R_a^{**} \cap E(S)$, then

$$f = ef = fe = e.$$

We denote the unique idempotent in R_a^{**} of S by a^+. Since S is a right-e wlpp semigroup, it follows that

$$aa^+ = a = a^+a, \qquad a \in S.$$

Lemma 1.2 *If S is a right-e wlpp semigroup, then $\mathcal{R}^{**}$ is a congruence on S.*

Proof. It is easy to see that the relation $\mathcal{R}^{**}$ is an equivalence. To show that $\mathcal{R}^{**}$ is compatible, let $(a, b) \in \mathcal{R}^{**}$ for $a, b \in S$. Then $a^+ = b^+$ by Lemma 1.1. Suppose that $(xac,\ yac) \in \mathcal{L}$ for any $x, y \in S^1$ and any $c \in S$. Since $(c, c^+) \in \mathcal{R}^{**}$, it follows that $(xac^+,\ yac^+) \in \mathcal{L}$. But $(xac^+,\ yac^+) = (xa^+ac^+,\ ya^+ac^+)$ and hence $(xa^+c^+a,\ ya^+c^+a) \in \mathcal{L}$. From $a^+ = b^+$ and $(a, b) \in \mathcal{R}^{**}$, we have $(xa^+c^+b,\ ya^+c^+b) \in \mathcal{L}$ and $(xb^+c^+b,\ yb^+c^+b) = (xb^+bc^+,\ yb^+bc^+) = (xbc^+,\ ybc^+) \in \mathcal{L}$. Again, from $(c, c^+) \in \mathcal{R}^{**}$, we have $(xbc,\ ybc) \in \mathcal{L}$.

Similarly, we can show that $(xbc,\ ybc) \in \mathcal{L}$ implies $(xac,\ yac) \in \mathcal{L}$. Hence, $(ac,\ bc) \in \mathcal{R}^{**}$. This shows that $\mathcal{R}^{**}$ is a right congruence on S.

Suppose that $(xca,\ yca) \in \mathcal{L}$. Since $(a, b) \in \mathcal{R}^{**}$, we obtain that $(xcb,\ ycb) \in \mathcal{L}$ and so $\mathcal{R}^{**}$ is a left congruence on S. Thus, we have proved that $\mathcal{R}^{**}$ is a congruence on S.

Lemma 1.3 *Suppose that S is a right-e wlpp semigroup. Then $(ab)^+ = a^+b^+$ for all $a, b \in S$.*

Proof. Let S be a right-e wlpp semigroup. we have $(a,\ a^+) \in \mathcal{R}^{**}$ and $(b,\ b^+) \in \mathcal{R}^{**}$. Then, by Lemma 1.2, $(ab,\ a^+b^+) \in \mathcal{R}^{**}$ for all $a, b \in S$ since $\mathcal{R}^{**}$ is a congruence on S. Thus $(ab)^+ = a^+b^+$ by Lemma 1.1.

Let S be a right-e wlpp semigroup. For all $a, b \in S$, we define a relation ρ by $a\rho b$ if and only if $a = fb$ for some $f \in E(b^+)$, where $E(b^+)$ is a rectangular band containing idempotent b^+.

Lemma 1.4 *Let S be a right-e wlpp semigroup and ρ be the above relation defined on S. Then ρ is a congruence on S.*

Proof. We now claim that ρ is an equivalent relation. Clearly, ρ is reflexive and symmetric. To show that ρ is transitive, we first prove that if $a\rho b$, then $E(a^+) = E(b^+)$ for any $a, b \in S$.