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Stable t-structures and Homotopy Category of Strongly Copure Projective Modules

Ma Xin and Zhao You-yi
(*College of Natural Sciences, Gansu Agricultural University, Lanzhou, 730070*)

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Abstract: In this paper, we study the homotopy category of unbounded complexes of strongly copure projective modules with bounded relative homologies $\mathcal{K}^{\infty,bscp}(\mathcal{SCP})$. We show that the existence of a right recollement of $\mathcal{K}^{\infty,bscp}(\mathcal{SCP})$ with respect to $\mathcal{K}^{-,bscp}(\mathcal{SCP})$, $\mathcal{K}_{scpac}(\mathcal{SCP})$ and $\mathcal{K}^{\infty,bscp}(\mathcal{SCP})$ has the homotopy category of strongly copure projective acyclic complexes as a triangulated subcategory in some case.
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1 Introduction and Preliminaries

As we know, triangulated categories play an important role on communicating algebraic reprensentability theory with algebraic geometry. The recollements and t-structures of triangulated categories introduced by Beilinson *et al.*[1] are the key factors in this process. Recollements of triangulated categories with the idea that $\mathcal{T}$ can be viewed as being "glued together" from $\mathcal{T}'$ and $\mathcal{T}''$. The canonical example of a recollement has $\mathcal{T}$, $\mathcal{T}'$ and $\mathcal{T}''$ equal to suitable derived categories of sheaves on spaces X, Z and U, where X is the union of the closed subspace Z and its open complement U. Miyachi[2] introduced the stable t-structure of triangulated categories, which relates the recollements.

In Section 2, we study some properties of strongly copure projective acyclic complexes. In Section 3, let $\mathcal{SCP}$ be the category of strongly copure projective R-modules, which are also called strongly P-projective modules. The notion is introduced by Mao[3], M is said to be strongly P-projective module if $\mathrm{Ext}^i_R(M,\, P) = 0$ for all projective left R-modules P,

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Foundation item: The NSF (11461060) of China.
E-mail address: maxin263@126.com (Ma X).

which is dual to the notion of strongly copure injective R-modules in [4]. So we call them strongly copure projective modules in this paper, which is a generalization of Gorenstein projective modules. We consider the relation of the homotopy category $\mathcal{K}^{\infty,bscp}(\mathcal{SCP})$ of unbounded complexes with bounded relative homologies, its subcategories $\mathcal{K}^{-,bscp}(\mathcal{SCP})$, and the homotopy category $\mathcal{K}_{scpac}(\mathcal{SCP})$ of strongly copure projective acyclic complexes. We show that the existence of a right recollement of the homotopy category $\mathcal{K}^{\infty,bscp}(\mathcal{SCP})$ and it has $\mathcal{K}_{scpac}(\mathcal{SCP})$ as a triangulated subcategory when R is an n-FC ring.

Next we recall some known notions and facts needed in the sequel.

Let R be a ring with unity and $\mathcal{C}(R)$ the category of complexes of R-modules. A complex X in $\mathcal{C}(R)$ is said to be bounded below if there exists an $N \in \mathbf{Z}$ such that $X^n = 0$ for $n \leq N$ and bounded above if there exists an $N \in \mathbf{Z}$ such that $X^n = 0$ for all $n \geq N$. A complex is bounded if it is both bounded above and bounded below. We define three full subcategories of $\mathcal{K}(R)$:

$\mathcal{K}^+(R)$: bounded below complexes;

$\mathcal{K}^-(R)$: bounded above complexes;

$\mathcal{K}^b(R)$: bounded complexes.

Let X be in $\mathcal{C}(R)$ and $n \in \mathbf{Z}$. We define another complex in $\mathcal{C}(R)$ by "shifting" n places to the left (writing cochain complexes with indices ascending to the right)

$$X[n]^p = X^{p+n}, \qquad \partial^p_{X[n]} = (-1)^n \partial^{p+n}_X.$$

If $f\colon X \longrightarrow Y$ is a morphism of complexes, then $f[n]^p = f^{p+n}$ defines a morphism of complexes $f[n]\colon X[n] \longrightarrow Y[n]$.

Let $f\colon X \longrightarrow Y$ be a morphism of complexes. $f\colon X \longrightarrow Y$ is a quasi-isomorphism if the morphism $H^n(f)\colon H^n(X) \longrightarrow H^n(Y)$ is an isomorphism of abelian groups for every $n \in \mathbf{Z}$. The mapping cone $\mathrm{Cone}(f)$ of f is defined for $n \in \mathbf{Z}$ by $(\mathrm{Cone}(f))^n = X^{n+1} \oplus Y^n$ with the differential

$$\partial^n_{\mathrm{Cone}(f)}\colon X^{n+1} \oplus Y^n \longrightarrow X^{n+2} \oplus Y^{n+1}, \qquad \partial^n_{\mathrm{Cone}(f)} = \begin{pmatrix} -\partial^{n+1}_X & 0 \\ u^{n+1} & \partial^n_Y \end{pmatrix}.$$

$f\colon X \longrightarrow Y$ is a quasi-isomorphism if and only if the mapping cone $\mathrm{Cone}(f)$ is acyclic.

The homotopy category $\mathcal{K}(R)$ is the category whose objects are complexes in $\mathcal{C}(R)$ and whose morphisms are homotopy equivalence classes of morphisms of complexes. That is, we begin with the abelian category $\mathcal{C}(R)$ and use the homotopy equivalence relation to divide the morphism sets up into equivalence classes. As we all know, $\mathcal{K}(R)$ is a triangulated category with triangulations.

2 Strongly Copure Projective Acyclic Complex

This section is to introduce and study the strongly copure projective acyclic complex.

Definition 2.1 *A complex X^* is strongly copure projective acyclic, or simply, $\mathcal{SCP}$-acyclic, if $\mathrm{Hom}_R(M, X^*)$ is acyclic for all strongly copure projective R-modules M.*