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Topological Tail Pressure for Asymptotically Sub-additive Continuous Potentials

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Abstract: We define the topological tail pressure and the conditional pressure for asymptotically sub-additive continuous potentials on topological dynamical systems and obtain a variational principle for the topological tail pressure without any additional assumptions.

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1 Introduction

In this paper, we call (X, T) a topological dynamical system (TDS for short) if (X, d) is a compact metric space, $T : X \to X$ is a surjective and continuous map with finite topological entropy.

Topological pressure is a generalization of topological entropy, the theory of topological pressure, variational principles, equilibrium states and related topics plays a fundamental role in statistical mechanics, ergodic theory, and dynamical systems (see [1]–[5]).

Ruelle[6] first introduced the notion of the topological pressure and presented the related variational principle for additive potentials of expansive maps on compact metric spaces. Later, Walters[7] generalized the result to the general continuous maps on compact metric

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spaces:

Let (X, T) be a TDS, $f : X \to \mathbf{R}$ be an arbitrary continuous function and $P(T, f)$ be the topological pressure of f. Then

$$P(T, f) = \sup\left\{h_\mu(T) + \int f \mathrm{d}\mu : \mu \in M(X, T)\right\},$$

where $M(X, T)$ denotes the space of all T-invariant measures on X and $h_\mu(T)$ denotes the measure-theoretical entropy of μ.

In [8], Cao *et al.* defined the topological pressure and set up a variational principle for sub-additive potentials on general compact metric spaces without any additional assumptions. Then Feng *et al.*[9] extended the result to the case of asymptotically sub-additive potentials:

Let (X, T) be a TDS and $\mathcal{F} = \{f_n\}_{n=1}^{\infty}$ be an asymptotically sub-additive potentials on (X, T). Denote by $P(T, \mathcal{F})$ the topological pressure of $\mathcal{F}$, then

$$P(T, \mathcal{F}) = \sup\{h_\mu(T) + \mathcal{F}_*(\mu) : \mu \in M(X, T)\},$$

where

$$\mathcal{F}_*(\mu) = \lim_{n\to\infty} \frac{1}{n} \int f_n \mathrm{d}\mu.$$

Li *et al.*[10] defined the tail pressure and established a variational principle for continuous transformations on compact metric spaces:

Let (X, T) be a TDS, $f : X \to \mathbf{R}$ be continuous, and $P(T, f)$ be the tail pressure of f. Then

$$P(T, f) = \sup\left\{u(\mu) + \int f \mathrm{d}\mu : \mu \in M(X, T)\right\},$$

where

$$u(\mu) = \lim_{k\to\infty} \widetilde{h_\infty - h_k}(\mu),$$

and $\mathcal{H} = (h_k)_{k\in\mathbf{N}}$ is an entropy structure defined on $M(X, T)$. Moreover, the supremum can be achieved on the closure of the ergodic measures. When $f = 0$, the definition is equivalent with the tail entropy defined in [11]. Ding *et al.*[12] generalized the result to the sub-additive potentials:

Let (X, T) be a TDS, $\mathcal{F} = \{f_n\}_{n\in\mathbf{N}}$ be a sub-additive potential, and $P^*(T, \mathcal{F})$ be the topological pressure of $\mathcal{F}$. Then

$$P^*(T, \mathcal{F}) = \sup\{u(\mu) + \mathcal{F}_*(\mu) \mid \mu \in M(X, T)\}.$$

Moreover, the supremum can be achieved on the closure of the ergodic measures. When $\mathcal{F} = \{f\}$, the result is equivalent to that in [10].

The purpose of this paper is to extend above results to the case of asymptotically sub-additive potentials. We give a definition of the tail pressure for asymptotically sub-additive potentials, and establish a variational principle which implies the relation between the tail pressure and the tail entropy function.

We first introduce the definition of asymptotically sub-additive potentials before formulating our results, we take it from [9].