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The Expression of Slow-growing Meromorphic Functions Sharing One Value CM with Their Derivatives

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Abstract: In this paper, we study the relations between meromorphic functions and their derivatives with one shared values, and obtain a concrete expression of meromorphic functions of zero order that share one value CM with their derivatives by a new method. Our main result is the supplementary of a related result due to Li and Yi (Li X M, Yi H X. Uniqueness of meromorphic functions sharing a meromorphic function of a small order with their derivatives. *Ann. Polon. Math.*, 2010, **98**(3): 201–219).

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1 Introduction and Main Results

A meromorphic function always mean meromorphic in the whole complex plane in this paper. We say that two meromorphic functions $f(z)$ and $g(z)$ share a finite value a IM (ignoring multiplicities) provided that $f-a$ and $g-a$ have the same zeros. If $f-a$ and $g-a$ have the same zeros with the same multiplicities, then we say $f(z)$ and $g(z)$ share the value a CM (counting multiplicities). We assume that the reader is familiar with the fundamental concepts of Nevanlinna's value distribution theory, e.g., [1] and [2], and in particular with the most usual of its symbols: $m(r,\ f)$, $N(r,\ f)$, $T(r,\ f)$ and so on.

Brück[3] seems to be the first one who considered the relations between meromorphic

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functions and their derivatives concerning one shared value, and presented the following well-known conjecture in 1996.

Brück Conjecture[3] *Let f be a nonconstant entire function such that the hyper-order $\rho_2(f)$ of f is not a positive integer and $\rho_2(f) < \infty$. If f and f' share a finite value a* CM, *then $\dfrac{f'-a}{f-a} = c$ for some nonzero constant c.*

Here, $\rho_2(f) = \limsup\limits_{r\to\infty} \dfrac{\log^+ \log^+ T(r, f)}{\log r}$.

Brück[3] showed that the conjecture is true when $a = 0$, and Gundersen and Yang[4] proved the case that f is of finite order. The case $\rho(f) = \infty$ and $\rho_2(f) < \dfrac{1}{2}$ was be proved by Chen and Shou[5], where $\rho(f) = \limsup\limits_{r\to\infty} \dfrac{\log^+ T(r,\ f)}{\log r}$.

Although the Brück's Conjecture is still open by now, it has induced many papers to investigate the related uniqueness problems which are analogue to this conjecture.

In 1999, Yang[6] proved the following result.

Theorem 1.1[6] *Let f be a nonconstant entire function of finite order, a be a finite nonzero constant, and k be a positive integer. If f and $f^{(k)}$ share a CM, then $f^{(k)} - a = c(f - a)$ for some nonzero complex number c.*

Chang and Zhu[7] replaced the shared-value by a meromorphic function and proved the next result.

Theorem 1.2[7] *Let f and a be two meromorphic functions such that f and a have finitely many poles, and such that f and a have no common poles. If $f - a$ and $f' - a$ share* 0 *CM, and if $\rho(a) < \rho(f) < \infty$, then $f' - a = c(f - a)$ for some nonzero constant c.*

Later, Li and Yi[8] improved Theorem 1.2 to the next result.

Theorem 1.3 ([8], Theorem 1.2) *Let f and $a(\not\equiv 0)$ be two meromorphic functions such that f and a have finitely many poles, and such that f and a have no common poles. If $\rho(a) < \rho(f) < \infty$ and if $f - a$ and $f^{(k)} - a$ share* 0 *CM, where $k(\geq 1)$ is an integer, then $f^{(k)} - a = c(f - a)$ for some nonzero constant c.*

In [7] and [8], the authors showed by an example that the condition "$\rho(a) < \rho(f)$" cannot be replaced by "$\rho(a) = \rho(f)$". Let $f(z) = e^{2z} - (z - 1)e^z$ and $a(z) = e^{2z} - ze^z$. Then we can verify that $f - a$ and $f' - a$ share 0 CM and $f' - a = e^z(f - a)$.

However, by an obvious observation, we can find that the order of f in Theorems 1.2 and 1.3 is strictly greater than 0. Thus, it is interesting to ask:

Question 1.1 Suppose that f is a meromorphic function with at most finitely many poles such that $\rho(f) = 0$. What can be said if f and $f^{(k)}$ share a finite nonzero complex value a CM?