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On Meromorphic Solutions of Nonlinear Complex Differential Equations

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Abstract: Applying the Nevanlinna theory of meromorphic function, we investigate the non-admissible meromorphic solutions of nonlinear complex algebraic differential equation and gain a general result. Meanwhile, we prove that the meromorphic solutions of some types of the systems of nonlinear complex differential equations are non-admissible. Moreover, the form of the systems of equations with admissible solutions is discussed.

Key words: non-admissible solution, complex differential equation, value distribution, meromorphic function

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1 Introduction and Main Results

Let $w(z)$ be meromorphic in the complex plane, and we use the standard notations of Nevanlinna theory of meromorphic function (see [1]–[2]), which are also introduced as follows for convenient to read:

$$N(r, w) := \int_0^r \frac{n(r, w) - n(0, w)}{t} \mathrm{d}t + n(0, w) \log r, \qquad \text{(Counting function)}$$

$$m(r, w) := \frac{1}{2\pi} \int_0^{2\pi} \log^+ |w(r\mathrm{e}^{\mathrm{i}\varphi})| \mathrm{d}\varphi, \qquad \text{(Proximity function)}$$

$$T(r, w) := m(r, w) + N(r, w), \qquad \text{(Characteristic function)}$$

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$$\bar{N}(r, w) := \int_0^r \frac{\bar{n}(r, w) - \bar{n}(0, w)}{t} \mathrm{d}t + \bar{n}(0, w) \log r, \qquad \text{(Reduced counting function)}$$

where $n(r, w)$ counts the number of the pole of $w(z)$ in $|z| \le r$, each pole according to its multiplicity, and $\bar{n}(r, w)$ counts the number of distinct poles of $w(z)$ in $|z| \le r$. We call an error term and denote by $S(r, w)$ any quantity satisfying

$$S(r, w) := o\{T(r, w)\}$$

as $r \to \infty$, possibly outside a set of r of finite linear measure.

Some authors investigated the existence of meromorphic solutions of algebraic differential equations and obtained many meaningful results (see [3]–[11]). Gackstatter and Laine[3] considered the following differential equation

$$(w')^n = \sum_{j=1}^{m} a_j(z) w^j \qquad (m \ge 1). \tag{1.1}$$

They gave the conjecture that: the differential equation (1.1) does not possess any admissible solutions if $1 \le m \le n-1$. For simplicity, we denote $S(r) = \sum\limits_{j=1}^{m} T(r, a_j)$. w is called an admissible solution of (1.1) if $S(r) = o(T(r, w))$.

He and Laine[6] proved the conjecture in [3]. While they obtained the following theorem:

Theorem A[6] The differential equation (1.1) does not possess any admissible solutions for $1 \le m \le n-1$.

Gao[7] investigated the following high-order differential equation:

$$\left(\frac{\Omega(z,w)}{P(z,w)}\right)^m = a_p(z) w^p + \sum_{i=0}^{s} a_i(z) w^i, \qquad a_p \not\equiv 0,\ a_s \not\equiv 0,\ s < p, \tag{1.2}$$

where

$$\Omega(z,w) = \sum_{(k)} c_k(z) (w)^{k_0} (w')^{k_1} (w'')^{k_2} \cdots (w^{(n)})^{k_n}$$

and

$$P(z,w) = (w)^{i_0} (w')^{i_1} (w'')^{i_2} \cdots (w^{(n)})^{i_n}.$$

He obtained the result as below:

Theorem B[7] Let $w(z)$ be a meromorphic solution of (1.2). If

$$\bar{N}(r, w) + \bar{N}\left(r, \frac{1}{w}\right) = S(r, w), \qquad p - s - \frac{p}{m} > 0,$$

then $w(z)$ is a non-admissible solution.

Remark 1.1 From the proof of Theorem B, we know that $p - s - \dfrac{p}{m} > 0$, if $p < m$. Therefore, the condition $p \ge m$ can be removed (see [7]).

Motivated by [7], we reinvestigate meromorphic solution of (1.2) when

$$\bar{N}(r, w) + \bar{N}\left(r, \frac{1}{w}\right) \ne S(r, w).$$

More precisely, one of our results can be stated as follows:

Theorem 1.1 *Let $w(z)$ be a meromorphic solution of (1.2). If*

$$4\bar{N}(r, w) + \bar{N}\left(r, \frac{1}{w}\right) = (\lambda + o(1)) T(r, w), \quad p - s - \frac{p}{m} > \lambda, \qquad \lambda \ge 0,$$

then $w(z)$ is a non-admissible solution.