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Weakly Disk-busting Curves in the Boundary of a Compression Body

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Abstract: Let V be a nontrivial compression body which is not simple. An essential simple closed curve J in $\partial_+ V$ is called weakly disk-busting if $\partial_+ V - J$ has only one compressing disk up to isotopy. In this paper, we give an upper bound of the diameter of the image of boundaries of essential disks in V under any projection determined by a weakly disk-busting curve. Moreover, we give a sufficient condition for the handle additions to be boundary irreducible.

Key words: handle addition, curve complex, subsurface projection, weakly disk-busting

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1 Introduction

Let M be a compact and orientable 3-manifold and F a component of ∂M. For an essential simple closed curve J in F, let M_J be the manifold obtained by attaching a 2-handle to M along J and filling in the resulting possible 2-sphere with a 3-ball. Such an operation is called a handle addition to M along J. If ∂M_J is incompressible in M_J, we say that the handle addition (as well as, J) is ∂-irreducible.

Przytycki[1] first gave a sufficient condition for a handle addition to a handlebody to be ∂-irreducible, Jaco[2] then generalized it to obtain the well-known handle addition theorem. Since then many generalizations have been given (see [3]–[6]).

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In [7], Masur and Minsky introduced the subsurface projection to study the structure of the curve complex. They showed the diameter of the image of a geodesic under the projection is bounded by a constant $\mathcal{M}$, and $\mathcal{M}$ only depends on the genus of the surface. Li[8], Masur and Schleimer[9] independently gave an estimation of the diameter of the image of the essential disks in a compact boundary reducible 3-manifold under the projection.

Let V be a nontrivial compression body which is not simple. An essential simple closed curve J in $\partial_+ V$ is called weakly disk-busting if $\partial_+ V - J$ has only one compressing disk up to isotopy. In this paper, we give an upper bound of the diameter of the image of boundaries of essential disks in V under any projection determined by a weakly disk-busting curve. Moreover, we give a sufficient condition for the handle additions to be boundary irreducible, which can be seen as an extension of Theorem 2.2 of [4].

The article is organized as follows. In Section 2, we review some necessary preliminaries. In Section 3, we give the main results and proofs.

2 Preliminaries

Let M be a 3-manifold and F a properly embedded surface which is not a 2-sphere. F is called compressible if F is a disk parallel to the boundary of M or there exists a disk D such that $D \cap F = \partial D$ and ∂D is essential in F. In the second case, we call D a compressing disk for F.

Suppose that F is a compact orientable surface of genus at least 1. The curve complex of F, first introduced by Harvey[10], is defined as follows: each vertex is the isotopy class of an essential simple closed curve in F and $(k+1)$ vertices determine a k-simplex if they can be realized by pairwise disjoint curves. When F is a torus or once-punctured torus, the curve complex of F, defined by Masur and Minsky[7], is the complex whose vertices are isotopy classes of essential simple closed curves in F, and $(k+1)$ vertices determine a k-simplex if they can be realized by curves which mutually intersect in only one point. Denote the curve complex of F by $C(F)$.

For any two vertices α, β in $C(F)$, the distance between α and β, denoted by $d_{C(F)}(\alpha, \beta)$, is defined to be the minimal number of 1-simplices in all possible simplicial paths connecting α to β. The simplicial path realizes the distance between α and β is called a geodesic. Let A and B be any two sets of vertices in $C(F)$. The diameter of A, denoted by $\mathrm{diam}_{C(F)}(A)$, is defined to be $\max\{d(x, y) \mid x, y \in A\}$. The distance between A and B, denoted by $d_{C(F)}(A, B)$, is defined to be $\min\{d(x, y) \mid x \in A, y \in B\}$.

Let F be a compact orientable surface of genus at least 1 with nonempty boundary. Denote the arc and curve complex of F by $AC(F)$. Vertices of $AC(F)$ are isotopy classes of essential arcs or curves in F and $(k+1)$ vertices determine a k-simplex if they can be represented by pairwise disjoint arcs or curves. The distance between two vertices α, β, denoted by $d_{AC(F)}(\alpha, \beta)$, is defined to be the minimal number of 1-simplices in a simplicial path jointing α to β over all such possible paths.

Let F' be a subsurface of F such that each component of $\partial F'$ is essential in F. By