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Explicit Multi-symplectic Method for a High Order Wave Equation of KdV Type

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Abstract: In this paper, we consider multi-symplectic Fourier pseudospectral method for a high order integrable equation of KdV type, which describes many important physical phenomena. The multi-symplectic structure are constructed for the equation, and the conservation laws of the continuous equation are presented. The multi-symplectic discretization of each formulation is exemplified by the multi-symplectic Fourier pseudospectral scheme. The numerical experiments are given, and the results verify the efficiency of the Fourier pseudospectral method.
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1 Introduction

In 2002, Tzirtzilakis *et al.*[1] considered a class of water wave equation of KdV type (see [2]–[5])

$$u_t + u_x + \alpha u u_x + \beta u_{xxx} + \alpha^2 \rho_1 u^2 u_x + \alpha\beta(\rho_2 u u_{xxx} + \rho_3 u_x u_{xx}) = 0, \tag{1.1}$$

where ρ_i $(i = 1, 2, 3)$ are free parameters, and α, β are positive real constants. During recent years, more and more experts have paid great attention to (1.1) (see [6]–[8]). Long *et al.*[6] obtained some exact solutions for some special set of parameters of (1.1) by integral bifurcation method. To the best of the author's knowledge, no much work has been done to construct the numerical scheme for (1.1) till now. In this paper, we consider numerical method to study (1.1).

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Now we use the Weiss-Tabor-Carnevale (WTC) method to test the Painleve property of (1.1). If $u(x,\, t)$ is a solution of (1.1), then it can be expressed as a Laurent series in $\phi(x,\, t)$

$$u = \sum_{j=0}^{\infty} u_j \phi^{j+a}. \tag{1.2}$$

Inserting (1.2) into (1.1), a leading order analysis uniquely gives

$$a = -2, \qquad u_0 = \frac{6(\beta\rho_2 + 2\beta\rho_3)}{\alpha}\phi_x^2. \tag{1.3}$$

Substituting (1.2) and (1.3) into (1.1), we obtain

$$(j+1)(j-6)\left(j - \frac{j - 2(\rho_2 + 2\rho_3)}{\rho_3}\right) u_j = F_j(\phi_x,\, \phi_t,\, \cdots,\, u_0,\, u_1,\, \cdots,\, u_{j-1}). \tag{1.4}$$

When $\rho_2 = 2\rho_3$, (1.1) is integrable. In this paper, we consider multi-symplectic method to study the integrable equation of KdV type as follows:

$$u_t + u_x + \alpha u u_x + \beta u_{xxx} + \alpha^2 \rho_1 u^2 u_x + \alpha\beta\rho_2(u u_{xxx} + 2u_x u_{xx}) = 0. \tag{1.5}$$

Multi-symplectic methods have been used to compute multi-symplectic Hamiltonian PDEs from the view point of symplectic geometry (see [9]–[32]). A lot of multi-symplectic methods for designing symplectic integrators have appeared, including the box scheme (see [15]), Rung-kutta collocation scheme (see [17]), Preissmann scheme (see [18]), splitting scheme (see [28]), Fourier pseudospectral scheme (see [31]), etc. Simultaneously, some error analysis for multi-symplectic methods were presented and discussed in [13] and [14], etc. The numerical solutions of many nonlinear wave equations such as KdV equation (see [15] and [18]), Schrödinger equation (see [22], [23] and [29]), Klein-Gordon equation (see [21]) and KP equation (see [20]), etc. have been studied by the multi-symplectic methods. One the most popular multi-syplectic meyhod is Fourier pseudospectral method which has been successfully applied to modeling wave propagation. After elimination of some auxiliary variables, new multi-symplectic schemes have been obtained, and preserve very well the mass, energy and momentum in long-time evolution.

The outline of this paper is as follows. In Section 2, we give the multi-symplectic Hamiltonian structure of (1.5), and prove that the structure satisfies the multi-symplectic conservation law, local energy and momentum conservation laws. In Section 3, we give the multi-symplectic Fourier pseudospectral method and error estimates of local conservation laws. In Section 4, numerical experiments are given. Finally, a conclusion and some discussions are given in Section 5.

2 Multi-symplectic Structure for the High Order Wave Equation of KdV Type

By the multi-symplectic theory (see [9]–[11]), a wide range of conservative PDEs can be written as a multi-symplectic Hamiltonian system

$$\boldsymbol{M}\boldsymbol{z}_t + \boldsymbol{K}\boldsymbol{z}_x = \nabla_{\boldsymbol{z}} S(\boldsymbol{z}), \tag{2.1}$$

where $\boldsymbol{M},\, \boldsymbol{K} \in \mathbf{R}^{d\times d}$ are the skew-symmetric matrices, $\boldsymbol{z}(x,t)$ is the vector of state variables, $S : \mathbf{R}^n \to \mathbf{R}$ is a scalar-valued smooth function, $\nabla_{\boldsymbol{z}} S(\boldsymbol{z})$ denotes the gradient of the