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Exponential Convergence of Finite-dimensional Approximations to Linear Bond-based Peridynamic Boundary Value Problems

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Abstract: In this paper, we demonstrate that the finite-dimensional approximations to the solutions of a linear bond-based peridynamic boundary value problem converge to the exact solution exponentially with the analyticity assumption of the forcing term, therefore greatly improve the convergence rate derived in literature.

Key words: Peridynamics, exponential convergence, nonlocal boundary value problem, analytic function, finite-dimensional approximation

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1 Introduction

Peridynamics (see [1]–[2]) is motivated in aid of modeling the problems from continuum mechanics which involve spontaneous discontinuity forms in the motion of a material system. By replacing differentiation with integration, Peridynamic (PD) equations remain equally valid both on and off the points where a discontinuity in either displacment or its spatial derivatives is located. Also, under the assumption that particles separated by a finite distance can interact with each other, the PD model is a multiscale material model (see [3]–[5]) and naturally falls into the category of nonlocal models (see [6]–[7]). There have been many theoretical works on the mathematical foundation of PD models associated with boundary value problems (BVPs) (see [8]–[15]), where the well-posedness results of the corresponding PD systems were established. Meanwhile, to simulate PD models numerically, various discretization methods have been studied including finite difference, finite element, quadrature

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and particle based methods (see [10], [16]–[24]).

In [10], a functional analytical framework was built up to study the linear bond-based peridynamic equations associated with a particular kind of nonlocal boundary condition. Investigated were the finite-dimensional approximations to the solutions of the equations obtained by spectral method and finite element method; as a result, two corresponding general formulas of error estimates were derived. According to these formulas, however, one can only conclude that the optimal convergence is algebraic. This motivates us to think whether or not the convergence rate is improvable.

In this paper, we focus on the one-dimensional (1-D) stationary problems. Based on the theoretical framework developed in [10], firstly we show that analytic data functions produce analytic solutions with some appropriate restrictions on kernel function. Afterwards, we are able to prove that those finite-dimensional approximations will achieve exponential convergence under the analyticity assumption of the input data.

The paper is organized as follows. We briefly introduce the general PD equilibrium models at the begining of Section 2, after that, in the same section, we give the definition of the PD operator. The nonlocal BVP is formulated in Section 3, followed by a discussion about analyticity of solutions. We devote Section 4 to the expositions of our main results on exponential convergence of finite-dimensional approximations.

2 The 1-D Stationary Linear Bond-based PD Equation

Suppose that the body of a material occupies a reference configuration in a finite bar I. Let x denote the position of a particle in the reference configuration, $u(x)$ be its displacement as the motion of the body occurs and F, a functional of the displacement, be the pairwise force function per unit reference length due to interaction between particles. Then for a given point x, the force (per unit reference length) produced from interaction with other particles is computed by

$$\mathcal{L}_\delta u(x) = \int_{B_\delta(x)} F(u(x') - u(x),\, x' - x)\mathrm{d}x',$$

where $B_\delta(x) = \{x' \in I\colon |x' - x| < \delta\}$ is the neighborhood of x in the reference configuration with the horizon parameter $\delta > 0$ which is always considered very small. Assume that f is the external force per unit reference length, and mass density of the material is a unit constant for simplicity, then by the Newton's second law, the PD equilibrium equation is given by

$$\mathcal{L}_\delta u(x) + f(x) = 0,$$

i.e.,

$$-\mathcal{L}_\delta u(x) = f.$$

Definition 2.1 *Assume $u \in L^2$ defined on the interval $(-\delta, \pi + \delta)$ satisfies either*

$$\textit{odd in } (-\delta, \delta) \textit{ and } (\pi - \delta, \pi + \delta), \tag{2.1}$$