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Pullback Attractor of a Non-autonomous Model for Epitaxial Growth

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Abstract: In this paper, we consider a non-autonomous model for epitaxial growth. It is shown that a pullback attractor of the model exists when the external force has exponential growth.

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1 Introduction

Suppose that Ω is an open connected bounded domain in $\mathbf{R}^2$ with a smooth boundary $\partial\Omega$, g is an external forcing term with $g \in L^2_{\rm loc}(\mathbf{R}, L^2(\Omega))$. We are concerned with the following non-autonomous equation:

$$\frac{\partial u}{\partial t} = -\Delta^2 u - \nabla \cdot \left(\frac{\nabla u}{1 + |\nabla u|^2}\right) + g(x, t) \qquad \text{in } \Omega \times [\tau, \infty) \tag{1.1}$$

with the boundary value condition

$$u = \frac{\partial u}{\partial n} = 0 \qquad \text{on } \partial\Omega \times [\tau, \infty) \tag{1.2}$$

and the initial condition

$$u(x, \tau) = u_\tau(x) \qquad \text{in } \Omega. \tag{1.3}$$

Equation (1.1) describes the process growing of a crystal surface. Here, $u(x, t)$ denotes a displacement of height of surface from the standard level $u = 0$ at a position $x \in \Omega$. The term $-\Delta^2 u$ in (1.1) denotes a surface diffusion of adatoms which is caused by the difference of the chemical potential. In the meantime, $-\nabla \cdot \left(\frac{\nabla u}{1 + |\nabla u|^2}\right)$ denotes the effect of surface

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roughening. Johnson *et al.*[1] presented the equation (1.1) for describing the process of growing of a crystal surface on the basis of the BCF theory. Rost and Krug[2] studied the unstable epitaxy on singular surfaces by using (1.1) with a prescribed slope dependent surface current. The problems of stability and global attractor have been studied in various papers (see, e.g., [3]–[6]). Moreover, based on finite difference methods, finite element methods, the numerical solutions for (1.1) have been studied in some papers (see [7] and [8] and so on).

The pullback attractor for nonautonomous infinite dimensional dynamical systems is important for the study of PDEs, which has attracted much attention and made fast progress in recent years. Caraballo *et al.*[9] introduced the notion of the pullback $\mathcal{D}$-attractor for nonautonomous dynamical systems and gave a general method to prove the existence of pullback $\mathcal{D}$-attractor. Li and Zhong[10] proposed the concept of norm-to-weak continuous process and proved the existence of pullback attractors for the nonautonomous reaction-diffusion equation. The existence of pullback attractor for a nonautonomous modified Swift-Hohenberg equation when its external force has exponential growth was considered in [11]. Latterly, the pullback attractor for a generalized Cahn-Hilliard equation and a fourth-order parabolic equation modeling epitaxial thin film growth had been studied by Duan *et. al.*[12]–[13].

In this paper, we study the existence of pullback attractor for the problem (1.1)–(1.3) by employing the techniques in [10] and [11]. We give some basic results on the pullback attractor for non-autonomous dynamical systems.

Suppose that X is a complete metric space and $\{U(t, \tau)\} = \{U(t, \tau) : t \geq \tau, \tau \in \mathbf{R}\}$ is a two-parameter family of mappings acting on X.

Definition 1.1 [10]–[11] *$\{U(t, \tau)\}$ is said to be a norm-to-weak continuous process on X if*

- *$U(t, \tau) = U(t, s)U(s, \tau)$ for all $t \geq s \geq \tau$;*
- *$U(\tau, \tau) = Id$ (the identity operator) for all $\tau \in \mathbf{R}$;*
- *$U(t, \tau)x_n \to U(t, \tau)x$ if $x_n \to x$ in X.*

Suppose that $\mathcal{D}$ is a nonempty class of parameterized sets $D = \{D(t) : t \in \mathbf{R}\} \subset B(X)$, where $B(X)$ denotes the set of all bounded subsets of X.

Definition 1.2 [10]–[11] *$B = [B(t) : t \in \mathbf{R}\} \subset \mathcal{D}$ is a family of $\mathcal{D}$-absorbing set for the process $\{U(t, \tau)\}$ if for any $t \in \mathbf{R}$ and $D \subset \mathcal{D}$, there exists a $\tau_0(t, D) \leq t$ such that $\{U(t, \tau)\}D(\tau) \subset B(t)$ for all $\tau \leq \tau_0(t, D)$.*

Definition 1.3 [10]–[11] *A process $\{U(t, \tau)\}$ is said to be a satisfying condition (PDC) if for any fixed $t \in \mathbf{R}$, $D \in \mathcal{D}$ and any $\varepsilon > 0$, there exists a $\tau_0(t, D) \leq t$ and a finite dimensional subspace X_1 of X such that $P\left(\bigcup_{\tau \leq \tau_0} U(t, \tau)D(\tau)\right)$ is bounded, and*

$$\left\|(I - P)\left(\bigcup_{\tau \leq \tau_0} U(t, \tau)x\right)\right\|_X \leq \varepsilon, \qquad x \in D(\tau),$$

where $P : X \to X_1$ is a bounded projector.