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Coefficient Estimates for a Subclass of Bi-univalent Strongly Quasi-starlike Functions

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Abstract: The aim of this paper is to establish the Fekete-Szegö inequality for a subclass of bi-univalent strongly quasi-starlike functions which is defined in the open unit disk. Furthermore, the coefficients a_2 and a_3 for functions in this new subclass are estimated.

Key words: bi-univalent function, bi-univalent strongly quasi-starlike function, coefficient estimate, subordination

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1 Introduction

Let H denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{+\infty} a_n z^n \tag{1.1}$$

which are analytic on the open unit disk $U = \{z\colon |z| < 1\}$.

Let S denote the subclass of H consisting of univalent functions in U. Also, let S^*, C and K denote, respectively, the well-known subclasses of H consisting of univalent functions which are starlike, convex and close-to-convex.

Further, let $S^*(\alpha)$ and $C(\alpha)$ be the subclasses of S consisting of starlike functions of order α and convex functions of order α respectively. Their analytic descriptions are

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$$S^*(\alpha)=\left\{f(z)\in H\,\middle|\,\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha,\ 0\le\alpha<1\right\},$$

$$K(\alpha)=\left\{f(z)\in H\,\middle|\,\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha,\ 0\le\alpha<1\right\}.$$

In 1933, Fekete and Szegö[1] showed that for $f\in S$ given by (1.1)

$$|a_3-\mu a_2^2|\le\begin{cases}3-4\mu, & \mu\le 0;\\ 1+2\exp\left\{\dfrac{-2\mu}{1-\mu}\right\}, & 0\le\mu<1;\\ 4\mu-3, & \mu\ge 1.\end{cases}$$

In [2], Liu defined the class $T(\beta)$: Let $f(z)\in H$, $0<\beta\le 1$. If

$$\left|\arg\frac{(zf'(z))'}{g'(z)}\right|\le\frac{\beta\pi}{2},\qquad z\in U,\ g(z)\in S^*,$$

then $f(z)\in T(\beta)$ is called strong quasi-starlike function of order β.

The well-known Koebe's one-quarter theorem asserts that every function $f\in S$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z))=z,\qquad z\in U$$

and

$$f(f^{-1}(\omega))=\omega,\qquad |\omega|<r_0(f),\ r_0(f)\ge\frac{1}{4}$$

where

$$f^{-1}(\omega)=\omega-a_2\omega^2+(2a_2^2-a_3)\omega^3-(5a_2^2-5a_2a_3+a_4)\omega^4+\cdots. \tag{1.2}$$

A function $f(z)\in H$ is called bi-univalent if only if both f and f^{-1} are normalized univalent functions on U. The class of bi-univalent functions is denoted by Σ. Lewin[3] first introduced the class of bi-univalent functions and showed that $|a_2|\le 1.51$. Since then, many different authors investigated the subclasses of the class of bi-univalent functions and obtained the upper bound of $|a_2|$ or $|a_n|$ $(n>2)$ (see [4]–[10]).

Let

$$g(z)=z+b_2z^2+b_3z^3+\cdots\in H$$

which is analytic on the open unit disk $U=\{z\colon |z|<1\}$. If the following conditions are satisfied:

$$\mathrm{Re}\left\{\frac{zg'(z)}{g(z)}\right\}>\alpha,\qquad z\in U$$

and

$$\mathrm{Re}\left\{\frac{wG'(w)}{G(w)}\right\}>\alpha,\qquad w\in U$$

where $0\le\alpha<1$, $G=g^{-1}$, then $g(z)$ is called the bi-univalent starlike analytic function of order α. The class of bi-univalent starlike analytic functions of order α is denoted by $S^*_\Sigma(\alpha)$.

Similarly, we defined a new class of analytic functions: Let $0<\beta\le 1$, $f(z)=z+\sum\limits_{n=2}^{+\infty}a_nz^n\in\Sigma$. If $g\in S^*_\Sigma(0)$ such that