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Partial or Truncated Sharing Values of Meromorphic Functions with Their Shifts

Lian Gui, Chen Jun-fan* and Cai Xiao-hua
(*Department of Mathematics, Fujian Normal University, Fuzhou, 350117*)

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Abstract: We mainly study the periodicity theorems of meromorphic functions having truncated or partial sharing values with their shifts, where meromorphic functions are of hyper order less than 1 and $N(r, f) = \alpha T(r, f)$ for some positive number α.

Key words: meromorphic function, periodicity, truncated sharing value, partial sharing value

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1 Introduction and Results

Throughout the paper, meromorphic functions always mean non-constant meromorphic functions in the complex plane. The whole paper uses the standard notation of Nevanlinna theory (see [1]–[3]), such as $T(r, f)$, $N(r, f)$, $m(r, f)$, $\bar{N}(r, f)$ and so on. For such a meromorphic function f, we denote by $S(f)$ the all quantities satisfying $S(r, f) = o(T(r, f))$, as r tends to infinity outside of a possible exceptional set of finite logarithmic measure and possibly different each time. Moreover, $S(f)$ contains constant functions and $\hat{S}(f)$ means $S(f)\bigcup\{\infty\}$. In addition, the order $\rho(f)$ and hyper order $\rho_2(f)$ of a meromorphic function are defined in turn as follows:

$$\rho(f) = \limsup_{r\to\infty} \frac{\log^+ T(r, f)}{\log r}, \qquad \rho_2(f) = \limsup_{r\to\infty} \frac{\log\log^+ T(r, f)}{\log r}.$$

Given $a \in \hat{S}(f)$, we denote by $\bar{E}(a, f)$ the set of zeros of $f(z) - a(z)$, that is,

$$\bar{E}(a, f) = \{z\colon f(z) - a(z) = 0\}.$$

We say that two meromorphic functions f and g share a IM (ignoring multiplicities) if $\bar{E}(a, f) = \bar{E}(a, g)$. Moreover, if $\bar{E}(a, f) = \bar{E}(a, g)$ and the multiplicities of the zeros are

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* **Corresponding author**.
E-mail address: guilian0533@163.com (Lian G), junfanchen@163.com (Chen J F).

pairwise the same, then we say f and g share a CM (counting multiplicities).

Furthermore, let $a \in \hat{S}(f)$ and k be a positive integer. We use $\bar{E}_{k)}\left(r,\ \frac{1}{f-a}\right)$ to denote the set of zeros of $f(z)-a(z)$ with multiplicities no greater than k and each zero is counted only once. Let $N_{k)}\left(r,\ \frac{1}{f-a}\right)$ be the counting function of zeros of $f(z)-a(z)$ whose multiplicities are no more than k and $\bar{N}_{k)}\left(r,\ \frac{1}{f-a}\right)$ be the reduced counting function correspondingly. Similarly, $\bar{E}_{(k}\left(r,\ \frac{1}{f-a}\right)$ denotes the set of zeros of $f(z)-a(z)$ with multiplicities no less than k ignoring multiplicities, $N_{(k}\left(r,\ \frac{1}{f-a}\right)$ denotes the corresponding counting function and $\bar{N}_{(k}\left(r,\ \frac{1}{f-a}\right)$ denotes the corresponding reduced counting function.

For two meromorphic functions f, g and a positive integer k, we say that a is a truncated sharing value of them if

$$\bar{E}_{k)}\left(r,\ \frac{1}{f-a}\right) = \bar{E}_{k)}\left(r,\ \frac{1}{g-a}\right)$$

or

$$\bar{E}_{(k}\left(r,\ \frac{1}{f-a}\right) = \bar{E}_{(k}\left(r,\ \frac{1}{g-a}\right).$$

Obviously, f and g share a IM also means

$$\bar{E}_{(1}\left(r,\ \frac{1}{f-a}\right) = \bar{E}_{(1}\left(r,\ \frac{1}{g-a}\right),$$

which shows that truncated sharing values are more general to some extent. We define that a meromorphic function f shares a partially with a meromorphic function g, if $\bar{E}(a,\ f(z)) \subseteq \bar{E}(a,\ g(z))$ and here a is called partial sharing value. It is also easy to see that f and g share a IM which means

$$\bar{E}(a,\ f(z)) \subseteq \bar{E}(a,\ g(z)) \quad \text{and} \quad \bar{E}(a,\ f(z)) \supseteq \bar{E}(a,\ g(z)).$$

It follows that partial sharing values are more general.

This paper generalize the definition, called the pseudo-deficiency put forward by Yang[2], by replacing the constant with the small function.

Definition 1.1 *Let $f(z)$ be a transcendental meromorphic function, $a \in \hat{S}(f)$ and k be a positive number. Then we define*

$$\delta_{k)}(a,\ f) = 1 - \limsup_{r\to\infty} \frac{\bar{N}_{k)}\left(r,\ \frac{1}{f-a}\right)}{T(r,\ f)}.$$

In 2011, Heittokangas *et al.*[4] obtained the following sufficient condition for periodicity of a meromorphic function of finite order from the view of sharing values.