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The Shephard Type Problems for General L_p-Centroid Bodies

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Abstract: In this paper, combining with the L_p-dual geominimal surface area and the general L_p-centroid bodies, we research the Shephard type problems for general L_p-centroid bodies.

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1 Introduction

Let $\mathcal{K}^n$ denote the set of convex bodies (compact, convex subsets with nonempty interiors) in Euclidean space $\mathbf{R}^n$. Let $\mathcal{K}^n_o$ and $\mathcal{K}^n_c$ respectively denote the set of convex bodies containing the origin in their interiors and the set of convex bodies whose centroids lie at the origin. Besides, for the set of star bodies (about the origin) and the set of star bodies whose centroids lie at the origin in $\mathbf{R}^n$, we write $\mathcal{S}^n_o$ and $\mathcal{S}^n_c$, respectively. Let S^{n-1} denote the unit sphere in $\mathbf{R}^n$ and $V(K)$ denote the n-dimensional volume of a body K. For the standard unit ball B in $\mathbf{R}^n$, its volume is written by $\omega_n = V(B)$.

The notion of centroid body was introduced by Petty[1]. In [2], for a compact set K, the centroid body, ΓK, of K is an origin-symmetric convex body whose support function is defined by

$$h(\Gamma K, u) = \frac{1}{V(K)} \int_K |u \cdot x| \mathrm{d}x, \qquad u \in S^{n-1}.$$

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The centroid body is one of the most important notions in the Brunn-Minkowski theory. In recent decades, the centroid bodies have attracted increasing attention (see [2] and [3]).

In 1997, Lutwak and Zhang[4] introduced the notion of L_p-centroid bodies. For each compact star-shaped (about the origin) K in $\mathbf{R}^n$ and real number $p \geq 1$, the L_p-centroid body, $\mathit{\Gamma}_p K$, of K is an origin-symmetric convex body whose support function is defined by

$$\begin{aligned} h(\mathit{\Gamma}_p K, u)^p &= \frac{1}{c_{n,p} V(K)} \int_K |u \cdot x|^p \mathrm{d}x \\ &= \frac{1}{c_{n,p}(n+p) V(K)} \int_{S^{n-1}} |u \cdot v|^p \rho(K, v)^{n+p} \mathrm{d}S(v), \qquad u \in S^{n-1}. \end{aligned} \tag{1.1}$$

Here

$$c_{n,p} = \frac{\omega_{n+p}}{\omega_2 \omega_n \omega_{p-1}},$$

and $\mathrm{d}S(v)$ denotes the standard spherical Lebesgue measure on S^{n-1}. Regarding the investigations of L_p-centroid bodies, we may refer to [5]–[14].

In 2005, Ludwig[15] introduced a function $\varphi_\tau : \mathbf{R} \to [0, +\infty)$ by

$$\varphi_\tau(t) = |t| + \tau t \tag{1.2}$$

with a parameter $\tau \in [-1, 1]$.

Based on L_p-centroid bodies and function (1.2), Feng *et al.*[16] defined a corresponding notion of general L_p-centroid bodies in [16]. For $K \in \mathcal{S}_o^n$, $p \geq 1$ and $\tau \in [-1, 1]$, the general L_p-centroid body, $\mathit{\Gamma}_p^\tau K$, of K is a convex body whose support function is defined by

$$\begin{aligned} h(\mathit{\Gamma}_p^\tau K, u)^p &= \frac{1}{c_{n,p}(\tau) V(K)} \int_K \varphi_\tau(u \cdot x)^p \mathrm{d}x \\ &= \frac{1}{c_{n,p}(\tau)(n+p) V(K)} \int_{S^{n-1}} \varphi_\tau(u \cdot v)^p \rho(K, v)^{n+p} \mathrm{d}S(v), \end{aligned} \tag{1.3}$$

where

$$c_{n,p}(\tau) = \frac{1}{2} c_{n,p} [(1+\tau)^p + (1-\tau)^p].$$

The normalization is chosen such that

$$\mathit{\Gamma}_p^\tau B = B, \qquad \tau \in [-1, 1]$$

and

$$\mathit{\Gamma}_p^0 K = \mathit{\Gamma}_p K.$$

For the more investigations of general L_p-centroid bodies, see [16]–[18].

Combined with L_p-mixed volume, Lutwak[19] gave the definition of L_p-geominimal surface area. For $K \in \mathcal{K}_o^n$, $p \geq 1$, the L_p-geominimal surface area, $G_p(K)$, of K is defined by

$$\omega_n^{\frac{p}{n}} G_p(K) = \inf \left\{ n V_p(K, Q) V(Q^*)^{\frac{p}{n}} : Q \in \mathcal{K}_o^n \right\}.$$

Here $V_p(M, N)$ denotes the L_p-mixed volume of $M, N \in \mathcal{K}_o^n$. When $p = 1$, $G_1(K)$ is just the classical counterpart which was introduced by Petty[20]. The L_p-geominimal surface area have got many results from these articles (see [21]–[24]).

According to the notion of L_p-geominimal surface area and L_p-dual mixed volume, Wang and Qi[25] introduced the definition of L_p-dual geominimal surface area. For $K \in \mathcal{S}_c^n$, $p \geq 1$,