

# Consequence Operators and Information Algebras

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**Abstract:** In this paper, the continuity of consequence operators is introduced. Relations between consequence operators and their associated information algebras are mainly explored. It is shown that, for any given consequence operator, the associated information algebra can be directly constructed without any additional conditions. Furthermore, if the consequence operators are continuous or compact, then the associated information algebras are continuous or compact.

**Key words:** information algebra, consequence operator, continuity, compactness

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## 1 Introduction

Information is considered as collections of distinct, abstract items or elements — pieces or bodies of information. Combination of pieces of information and focusing of information are two fundamental operations on information. The two operations must be subdued to a set of intuitive axioms. In this way an information algebra is formed which can be seen as generic algebraic structures for local computation and inference (see [1]). Some related studies show that the framework of information algebra covers a wide range of different instances from constraint systems, propositional logic to belief functions, relational algebra and others (see [1]–[3]).

Information algebra is a generic structure closely related with information processing. However, real computer can treat only “finite” information. This means that not every information can be exactly represented and processed on real computers. However “infinite” information may possibly be approximated by “finite” information and thus processed

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at least approximately. To model this situation some compactness conditions have to be introduced into the algebra, leading to compact information algebras (see [1], [3] and [4]). Continuous information algebra is the generalization of compact information algebra (see [5] and [6]).

In order to give a different way to represent information and information algebra, the concept of consequence operator is introduced. Consequence operator (see [2]) is described by a language and an entailment relation on sets of formulae. References [2] and [4] show that any given information algebra can induce corresponding consequence operator and in turn, the associated information algebra is obtained on the consequence operator provided that the consequence operator satisfies the properties of interpolation and deduction. This means that compact consequence operator induced corresponding compact information algebra, provided that consequence operator is compact and satisfies the theorem of interpolation and deduction. Complexity of interpolation and deduction makes the link between information algebra and consequence operator not clear. And so far the description of equivalent of interpolation and the deduction is not able to give.

Therefore, it is necessary to find a new way to induce the corresponding information algebra by consequence operator, and also the new method should ensure the corresponding relationship between compact information algebra and compact consequence operator is clear. To address the issue, in this paper we firstly decompose the operation of focusing by consequence operator. Based on it, a complete and direct characterization of the relationship between information algebra and consequence operator is given. Secondly, the continuous consequence operator is defined and the relationship between continuous information algebra and continuous consequence operator is given. At last, according to the above conclusions, the paper shows that the compactness can be preserved from consequence operator to its associated information algebra.

The main work and the organization of this paper are as follows. In Section 2, some basic notions and examples of consequence operator and information algebra are introduced. Furthermore, the concepts of continuous (compact) consequence operator and continuous (compact) information algebra are given. Section 3 shows that, for any given consequence operator, the associated information algebra can be constructed directly without any additional conditions. Section 4 demonstrates the correspondence from continuous (or compact) consequence operators to continuous (or compact) information algebras.

## 2 Preliminaries

In this part, some basic notations on information algebra and consequence operator that we use in the following discussion will be introduced.

**Definition 2.1**<sup>[1]</sup> *A tuple  $(\Phi, D)$  is a system with two operations  $\cdot$  and  $\pi$ , where  $\Phi$  is a set of elements and  $D$  is a lattice:*

- (1) *Combination:  $\Phi \times \Phi \rightarrow \Phi, (\varphi, \psi) \rightarrow \varphi \cdot \psi$ ;*